



## Realization of the Contextuality-Nonlocality Tradeoff with a Qubit-Qutrit Photon Pair

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(Received 6 November 2015; published 29 February 2016)

We report our experimental results on the no-disturbance principle, which imposes a fundamental monogamy relation on contextuality versus nonlocality. We employ a photonic qutrit-qubit hybrid to explore no-disturbance monogamy at the quantum boundary spanned by noncontextuality and locality inequalities. In particular, we realize the single point where the quantum boundary meets the no-disturbance boundary. Our results agree with quantum theory and satisfy the stringent monogamy relation thereby providing direct experimental evidence of a tradeoff between locally contextual correlations and spatially separated correlations. Thus, our experiment provides evidence that entanglement is a particular manifestation of a more fundamental quantum resource.

DOI: 10.1103/PhysRevLett.116.090401

**Introduction.**—Quantum systems exhibit a wide range of nonclassical and counterintuitive phenomena, such as Bell nonlocality [1,2] and Kochen and Specker contextuality [3–7]. Corresponding experimental tests [8–17] have been performed and support the necessity of quantum mechanics. Previous works on contextuality and nonlocality monogamy relations have identified tradeoffs between the violation of either noncontextuality inequality [18,19] or Bell inequality [20,21]. The relation between contextual correlations and nonlocal correlations has been studied recently [22]. It has been proven that the no-disturbance (ND) principle imposes monogamy relation between contextuality and nonlocality, and the quantum version of this monogamy relation is even more stringent.

We experimentally demonstrate the quantum monogamy relation between contextuality and nonlocality in a photonic qutrit-qubit system. The simplest noncontextuality inequality, Klyachko-Can-Binicioğlu-Shumovski (KCBS) inequality [23–27], is tested within a single qutrit system and the Clauser-Horne-Shimony-Holt (CHSH) inequality [28] is tested for entanglement of a two-party system including this qutrit system and another qubit system. We find that the violation of one inequality forbids the violation of the other. Our work provides the first experimental evidence of the existence of a fundamental monogamy relation between contextuality and nonlocality imposed by quantum theory. The experimental results imply that the quantum resource [29], which rules out KCBS noncontextuality, might be a particular form of entanglement [22].

**Idea.**—We demonstrate no-disturbance monogamy spanned by noncontextuality and locality inequalities, which was theoretically proposed by Kurzyński *et al.* in [22]. Consider a scenario with two spatial separated observers, Alice and Bob. Alice randomly chooses two compatible measurements from five measurements  $\{A_i\}$  ( $i = 1, \dots, 5$ ) and performs them on her system. Each two of  $A_i$  and  $A_{(i+1)\bmod 5}$  are compatible, whereas Bob chooses one of two incompatible measurements  $B_1, B_2$  and performs them on his system. Each measurement has two outcomes  $\pm 1$ . The compatibility relations among the seven measurements are illustrated in Fig. 1.

One can test contextuality on Alice's system via KCBS inequality [23]

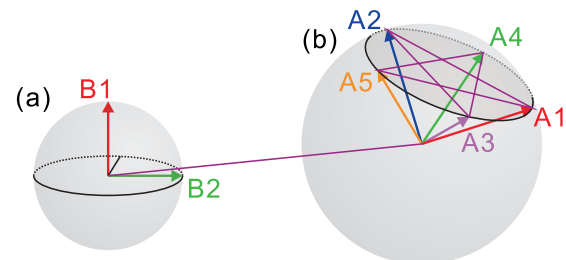


FIG. 1. (a) Representation of the measurement  $B_j$  ( $j = 1, 2$ ) in Bloch sphere. (b) Representation of the measurements  $A_i$  ( $i = 1, \dots, 5$ ) in three-dimensional space. The five directions are pairwise orthogonal, making the measurements pairwise compatible. The line connecting the center of two spheres denotes each  $B_j$  is compatible with  $A_i$ .

$$\begin{aligned} \kappa_A &= \langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle \\ &+ \langle A_5 A_1 \rangle \stackrel{\text{NCHV}}{\geq} -3. \end{aligned} \quad (1)$$

The violation implies the correlations cannot be described via a noncontextual hidden variable (NCHV) model. The maximal violation of the KCBS inequality is  $5 - 4\sqrt{5}$  [23], whereas the CHSH locality inequality [28]

$$\beta_{AB} = \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_4 B_1 \rangle - \langle A_4 B_2 \rangle \stackrel{\text{LHV}}{\geq} -2, \quad (2)$$

can be tested on the systems of Alice and Bob, and the violation implies the corresponding correlations cannot be described via a local hidden variable (LHV) model. The maximal violation of the CHSH inequality is  $-2\sqrt{2}$  [28]. The classical bounds of these two inequalities result from the noncontextuality assumption and can be violated due to the lack of a joint probability distribution. The maximal violation is bounded and the bound may result from the ND principle [18,30–32].

Based on the ND principle, the probabilities of outcomes of the measurement  $A_i$  do not depend on whether  $A_i$  is measured with  $A_{(i+1)\bmod 5}$  (compatible with  $A_i$ ). That is,  $p(a_i) = \sum_{a_{(i+1)\bmod 5}} P(a_i, a_{(i+1)\bmod 5})$ , where  $a_i$ s are the possible outcomes of  $A_i$ ,  $p(a_i)$  is a marginal distribution and  $P(a_i, a_{(i+1)\bmod 5})$  is a joint distribution.

The ND principle imposes a nontrivial tradeoff between the violations of CHSH and KCBS inequalities [22], i.e.,

$$\beta_{AB} + \kappa_A \stackrel{\text{ND}}{\geq} -5. \quad (3)$$

As we now show, according to the ND principle, only one of these inequalities can be violated at a time.

The monogamy relation holds in any theory satisfying the ND principle such as quantum theory. However, quantum theory imposes a more stringent monogamy relation between quantum contextual and nonlocal correlations. Consider a quantum mechanical implementation of the scenario in which Alice and Bob share a qutrit-qubit system. The corresponding basis states are  $\{|0\rangle, |1\rangle, |2\rangle\}$  and  $\{|0\rangle, |1\rangle\}$ , respectively. Five different types of measurements which are taken by Alice are in the Householder form  $A_i = 2|v_i\rangle\langle v_i| - 1$ , where  $i = 1, \dots, 5$ ,  $\langle v_i | v_{(i+1)\bmod 5} \rangle = 0$  and  $\mathbb{1}$  is the identity matrix. In particular, the state  $|v_i\rangle$  is assumed to be [22]

$$|v_i\rangle \propto \left[ \cos \frac{4\pi i}{5} |0\rangle + \sin \frac{4\pi i}{5} |1\rangle + \sqrt{\cos \frac{\pi}{5}} |2\rangle \right]. \quad (4)$$

In contrast, for CHSH scenario, Bob's observables are chosen to be two Pauli operators  $B_1 = Z$  and  $B_2 = X$ .

Quantum theory shows an additional monogamy relation between NCHV and LHV by restricting the possible values of  $(\beta_{AB}, \kappa_A)$  within a region in the parametric space

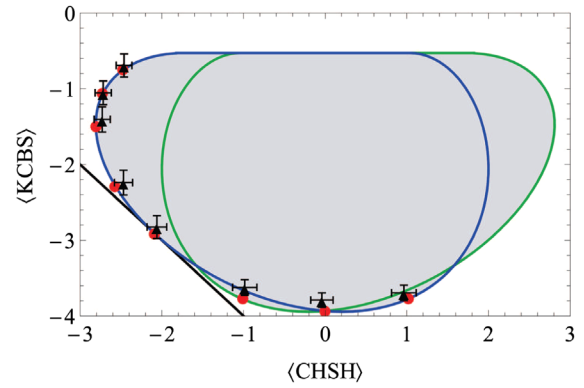


FIG. 2. The region spanned by the allowed average values of CHSH and KCBS operators  $\langle \text{CHSH} \rangle$  and  $\langle \text{KCBS} \rangle$  can be divided into two overlapping parts and bounded by the solid curves. The region bounded by the green curve is spanned by vectors which are linear combinations of  $\{|01\rangle, |10\rangle, |21\rangle\}$  and that bounded by the blue curve corresponds to the basis  $\{|00\rangle, |11\rangle, |20\rangle\}$ . Every quantum state produces a point inside the region. However, only the states in (5) can produce the points on the boundaries. The solid black straight line denotes the ND boundary. Experimental results of  $\langle \text{CHSH} \rangle$  and  $\langle \text{KCBS} \rangle$  are represented by the black triangles and compared to their theoretical predictions (red dots), producing the points on the boundary of the quantum region.

spanned by the value of these two inequalities. The more stringent monogamy relation makes the quantum region smaller than that imposed by the ND principle. Therefore, the boundary of the quantum region is more interesting, which can be produced by the states taking the form (unnormalized) [22]

$$\begin{aligned} |\psi_\phi^+\rangle &= f(\phi)|01\rangle + g(\phi)|10\rangle + |21\rangle, \\ |\psi_\phi^-\rangle &= f(\phi)|00\rangle + g(\phi)|11\rangle + |20\rangle, \end{aligned} \quad (5)$$

where

$$\begin{aligned} f(\phi) &\approx -0.05 + 0.15 \cot \phi - 0.57 \tan \phi, \\ g(\phi) &\approx 0.72 + 0.32 \cot \phi + 0.26 \tan \phi. \end{aligned}$$

The bounded quantum region is simulated and shown in Fig. 2.

The quantum boundary touches the ND boundary in a single point ( $\langle \text{CHSH} \rangle_{\text{th}} \approx -2.08$ ,  $\langle \text{KCBS} \rangle_{\text{th}} \approx -2.92$ , where  $\langle \text{CHSH} \rangle_{\text{th}}$  and  $\langle \text{KCBS} \rangle_{\text{th}}$  are the theoretical predictions of the average values of CHSH and KCBS operators, and equal to  $\beta_{AB}$  and  $\kappa_A$ , respectively, in quantum theory) by taking the choice of  $|\psi_\phi^\pm\rangle$  that minimizes the lower bound of  $\beta_{AB} + \kappa_A$ , which saturates the inequality (3): it becomes an equality. Even in the extreme case in which both of  $\kappa_A$  and  $\beta_{AB}$  are close to their classical bounds, the monogamy relation holds; i.e., the

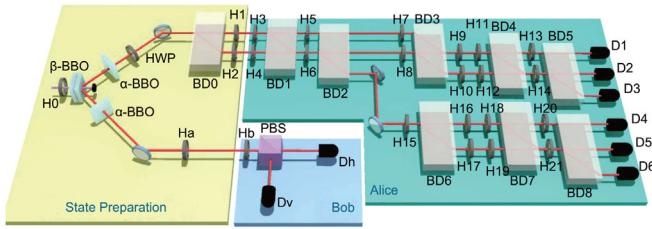


FIG. 3. Experimental setup. Entangled photons are generated via type-I SPDC. Two  $\alpha$ -BBO crystals and a following tilted HWP placed right after two joint  $\beta$ -BBO crystals are used to compensate the walk-off between photons with horizontal and vertical polarizations. For Alice, cascade setup for sequentially measuring  $A_i$  and  $A_{i+1}$  is used to test the KCBS inequality, whereas, to test the CHSH inequality,  $B_j$  is measured via standard polarization measurements using HWP (Hb) and PBS.

CHSH inequality is violated and the KCBS inequality is not violated.

*Experimental realization.*—To experimentally investigate the quantum monogamy relation between KCBS and CHSH inequalities, we produce the boundary of the quantum region in the parametric space spanned by the value of the two inequalities and, especially, the single point where the quantum boundary touches the ND boundary. We prepare the states of a qutrit-qubit system  $|\psi_{\phi}^{\pm}\rangle$  in Eq. (5) with the qubit encoded in the polarizations of photons and the qutrit encoded in both polarizations and spatial modes of photons.

As illustrated in Fig. 3, our experimental setup consists of three modules: state preparation, Alice’s measurement, and Bob’s measurement. In the state preparation module, entangled photons of 801.6 nm wavelength are generated in a type-I spontaneous parametric down-conversion (SPDC) process where two joint 0.5 mm-thick  $\beta$ -barium-borate ( $\beta$ -BBO) crystals are pumped by a cw diode laser with 90 mW of power [33,34]. The visibility of the entangled photonic state is larger than 95%. One of the photons as a qubit system is sent to Bob for his measurement. The basis states of the qubit system  $|0\rangle$  and  $|1\rangle$  are encoded in the horizontal and vertical polarizations of photons. The other is then split by a birefringent calcite beam displacer (BD) into two parallel spatial modes. The optical axis of the BD is cut so that vertically polarized photons are directly transmitted and horizontal photons undergo a lateral displacement of 3 mm into a neighboring mode. By employing the polarizations and spatial modes of a single photon, we can prepare the arbitrary state of a qutrit. The basis states  $|0\rangle$ ,  $|1\rangle$ , and  $|2\rangle$  are encoded in the horizontal polarization of the photon in the upper mode, the lower mode, and the vertical polarization of the photon in the upper mode, respectively. The state of the qutrit-qubit system is prepared in

$$\sin 2\theta_0 \sin 2\theta_1 |01\rangle - \cos 2\theta_0 |10\rangle - \sin 2\theta_0 \cos 2\theta_1 |21\rangle,$$

which equals the state  $|\psi_{\phi}^{\pm}\rangle$ , with

$$\theta_1 = -\frac{1}{2} \arctan f(\phi),$$

$$\theta_0 = \frac{1}{2} \arctan \frac{f(\phi) + 1}{g(\phi)(\cos 2\theta_1 - \sin 2\theta_1)}.$$

The parameter  $\phi$  can be adjusted via tuning the setting angles  $\theta_0$  and  $\theta_1$  of the half-wave plates (HWPs) before  $\beta$ -BBO (H0) and after BD0 (H1) (H2 is always set at  $45^\circ$  and used as an optical compensator). Similarly, the state  $|\psi_{\phi}^{-}\rangle$  can be generated by applying Ha at  $45^\circ$  on Bob’s qubit, and choosing the proper angles  $\theta_0$  and  $\theta_1$ . The angles of the HWPs for state preparation are listed in the Supplemental Material [35].

To measure Alice’s observables  $A_i$  and their correlations, we use cascaded Mach-Zehnder interferometers in three steps [11,36–38]. The first step is to realize the measurement of  $A_i$  with four HWPs (H3–6) and two BDs (BD1–2). The angles of H3 and H6 are chosen properly so that the photons which are in the state  $|v_i\rangle$  corresponding to eigenvalue 1 (–1) are mapped to be in horizontally (vertically) polarized mode after H6. The HWPs H4 and H5 can be tilted to fine tune the phase difference between the two arms of the interferometers.

Measuring  $A_i A_{i+1}$  requires two sequential measurements on the same photon. Since the single-observable measuring devices map its eigenstates to a fixed spatial path and polarization, with HWPs and BDs, we can recreate the corresponding eigenstates of  $A_i$  for further measurement  $A_{i+1}$  in the second step [37]. Two outcomes of  $A_i$  are each directed into identical but separated devices, i.e., H7–10 and BD3 for recreating the eigenstate corresponding to 1, and H15–17 and BD6 for recreating the eigenstate corresponding to –1.

In the third step, we use the same interferometers as in the first step to measure  $A_{i+1}$ . Two identical  $A_{i+1}$  measuring devices (one is constructed by H11–14, BD4–5, and another by H18–21, BD7–8) are built, each of which is connected to the corresponding output port of the measuring device of  $A_i$ . The angles of the HWPs for Alice’s measurements are listed in the Supplemental Material [35]. The outcomes of the measurement  $A_i A_{i+1}$  are given by the responses of the detectors (D1–6) and the assignments of the outcomes are shown in the Supplemental Material [35].

Though, theoretically, the violation of the inequality does not depend on the order of the measurements, tests of correlations in quantum contextuality in different orders have been considered in previous experiments [26,27]. In our experiment, a test of each correlation in the KCBS inequality in every possible order can also be implemented by rotating the setting angles of HWPs (H3–H21), such as in both  $A_i A_{i+1}$  and  $A_{i+1} A_i$  [35]. Further experiments to perform the measurements in different orders would be interesting.

For Bob, the measurement of observable  $B_j$  is a standard polarization measurement using HWP (Hb) and polarizing beam splitter (PBS). As we have mentioned before, Ha is

TABLE I. The experimental results of the average values of CHSH and KCBS operators for eight input states. Error bars indicate the statistical uncertainty.

State	$\phi$ (rad)	$\langle \text{CHSH} \rangle_{\text{ex}}$	$\langle \text{KCBS} \rangle_{\text{ex}}$
$ \psi_{\phi}^+\rangle$	-0.66	0.966(151)	-3.695(102)
$ \psi_{\phi}^-\rangle$	-0.66	-0.986(149)	-3.623(104)
$ \psi_{\phi}^+\rangle$	-0.48	-0.041(137)	-3.791(96)
$ \psi_{\phi}^+\rangle$	-0.27	-2.061(120)	-2.826(151)
$ \psi_{\phi}^+\rangle$	-0.22	-2.470(111)	-2.239(164)
$ \psi_{\phi}^+\rangle$	-0.16	-2.730(99)	-1.406(166)
$ \psi_{\phi}^+\rangle$	-0.12	-2.718(99)	-1.054(156)
$ \psi_{\phi}^+\rangle$	-0.08	-2.462(96)	-0.693(153)

used to prepare the state  $|\psi_{\phi}^-\rangle$ , whereas Hb at  $0^\circ$  or  $22.5^\circ$  is used to map the eigenstate of the observable  $B_1$  or  $B_2$  corresponding to the eigenvalue 1 ( $-1$ ) into the horizontally (vertically) polarizing state. The photons are detected by Dh and Dv right after the PBS.

For the photon detection, we only register the coincidence rates between the detectors (single-photon avalanche photodiodes with a 7 ns time window) of Alice and Bob. For each measurement, we record clicks for 20 s, and the total coincidence counts are  $\sim 2000$ . To test KCBS inequality, the correlation  $\langle A_i A_{i+1} \rangle$  is constructed from four measured joint probabilities  $P(A_i = \pm 1, A_{i+1} = \pm 1)$ . Similarly, we can evaluate the value of  $\beta_{AB}$  for the CHSH inequality with the correlation  $\langle A_i B_j \rangle$  which is constructed from the measured joint probability  $P(A_i = \pm 1, B_j = \pm 1)$ . All the joint probabilities can be read out from the coincidence between the certain detectors of Alice and Bob [35].

Photon loss opens up a detection efficiency loophole in our experiment. Thus, a fair-sampling assumption is taken, which assumes the event selected out by the photonic coincidence is an unbiased representation of the whole sample [35].

We produce eight points on the quantum boundary corresponding to eight different input states. The experimental results on the average values of the CHSH and KCBS operators are shown in Fig. 2 and Table I [35]. It is clear that the inequality (3) is always satisfied in experiment, and the violation of either KCBS or CHSH inequality forbids the violation of the other, in agreement with the quantum theory predictions. Especially, our results show the inequality (3) is tight; i.e., there is a state  $|\psi_{\phi}^+\rangle$  with  $\phi = -0.27$  for which the inequality becomes an equality. We present the measured values  $\langle \text{CHSH} \rangle_{\text{ex}} = -2.061 \pm 0.120$ ,  $\langle \text{KCBS} \rangle_{\text{ex}} = -2.826 \pm 0.151$  in the single point where the quantum boundary touches the ND boundary and the inequality becomes an equality; i.e.,  $\beta_{AB} + \kappa_A = -5$  is satisfied within error bars. We define

$$d_t = \frac{1}{\Delta_t} |\langle t \rangle_{\text{th}} - \langle t \rangle_{\text{ex}}|, \quad (6)$$

( $t$  representing CHSH and KCBS, and  $\Delta$  denoting the standard deviation) as a figure of merit to evaluate the quality of experimental demonstration. For the contact point,  $d_{\text{CHSH}} = 0.175$  and  $d_{\text{KCBS}} = 0.609$  are small for our experiment, indicating a successful experimental demonstration and providing strong evidence for the validity of a stringent monogamy relation between contextuality and nonlocality imposed by quantum theory.

*Conclusion.*—The monogamy of contextuality and monogamy of nonlocality have been studied, respectively. The fact that the origin of Bell inequalities and contextual inequalities is the existence of joint probability distributions naturally raises the question as to whether similar monogamy relations exist between contextual correlations and nonlocal correlations. Our experiment provides an answer to this question. We observe the fundamental monogamy relation between contextuality and nonlocality in a photonic qutrit-qubit system and show the first experimental evidence of a tradeoff between locally contextual correlations and spatially separated correlations imposed by quantum theory. The existence of the monogamy relation suggests the existence of a quantum resource of which entanglement is a particular form [22]. The resource required to violate KCBS inequality can be transformed into entanglement which consumes to violate CHSH inequality. Our experiment opens the door to experimentally observing other interesting phenomena such as quantum nonlocality based on local contextuality [39,40], and sheds new light for further explorations of this quantum resource. Furthermore, our results suggest that monogamy relations between different types of correlations might be ubiquitous in nature and pave the way for further research on these monogamy relations.

This work has been supported by National Natural Science Foundation of China, Grants No. 11174052, No. 11474049, and No. 61275122, the CAST Innovation fund, China's 1000 Talent Plan, AITF, and NSERC.

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