Focus on quantum tomography

2013 New J. Phys. 15 125020

(http://iopscience.iop.org/1367-2630/15/12/125020)

New Journal of Physics
The open access journal for physics

EDITORIAL

Focus on quantum tomography

K Banaszek[1], **M Cramer**[2,4] **and D Gross**[3]
[1] Faculty of Physics, University of Warsaw, ulica Hoza 69, P-00-681 Warsaw, Poland
[2] Institut für Theoretische Physik, Universität Ulm, Albert-Einstein Allee 11, D-89069 Ulm, Germany
[3] Institute for Physics, University of Freiburg, Rheinstrasse 10, D-79104 Freiburg, Germany
E-mail: marcus.cramer@uni-ulm.de

New Journal of Physics **15** (2013) 125020 (4pp)
Received 21 October 2013
Published 13 December 2013
Online at http://www.njp.org/
doi:10.1088/1367-2630/15/12/125020

Abstract. Quantum tomography has come a long way from early reconstructions of Wigner functions from projections along quadratures to the full characterization of multipartite systems. Now, it is routinely carried out in a wide variety of systems. And yet, many fundamental questions remain unanswered. In recent years, a spate of radical new experimental, theoretical and mathematical developments have occurred. The appeal of the subject lies largely in the breadth of techniques that must be brought together in order to fully understand the problem. This 'focus on' collection provides a platform for facilitating the exchange of ideas between the different communities involved in this process.

The ability to completely characterize the state and dynamics of a quantum system through physical measurements is an essential element in the emerging field of quantum technologies. Owing to extensive early research on the reconstruction of Wigner functions from their projections along a collection of quadratures [1, 2], this task is now commonly known as quantum tomography. Theoretical work on this problem dates back at least to the 1970s, and experimental implementations are routinely carried out in a wide variety of systems—in this

---

[4] Author to whom any correspondence should be addressed.

collection alone, the reader will encounter characterizations of single-photon and continuous-variable states and polarization transformations [3–9], cavity fields [10], atomic ensembles [11–13], trapped ions [14] and of optical detectors [15–17].

Despite this success, many fundamental questions remain unanswered. What is more, a variety of new experimental and theoretical developments have given rise to a substantial surge of research activity in this area in recent years. The most obvious development is the tremendous progress in controlling large, highly accessible quantum systems composed, for example, of trapped ions [18]. In these experimental platforms, measurements of arbitrary observables on individual systems can be performed with great accuracy. As a consequence, the bottleneck limiting further progress in estimating the states of such systems has shifted from physical controllability to the problem of handling the massive quantity of data resulting from the exponential scaling of the number of parameters describing quantum many-body states. This curse of dimensionality renders any naive approach to quantum tomography manifestly impossible, even for moderately large systems. Ultimately, this scaling problem cannot be overcome, it is a necessary feature of quantum processing devices outperforming the classical. It turns out, however, that the boundary where classical methods fail can be pushed significantly if non-trivial structural information on the quantum systems under consideration is utilized. We list a few examples: due to either physical reasons (low temperature) or 'engineering' reasons (in which one aims to prepare a pure state, which is commonly the case), states encountered in the laboratory are often fairly pure in the sense that their effective rank is small. This allows for state reconstruction with a square root improvement [14, 19–22]. The state under consideration may be the ground or thermal state of a local Hamiltonian, which are known to be well approximated by matrix product states and operators [23], allowing for a reconstruction that may be achieved with linearly many measurement settings [24–27]. A state may have been prepared such that it is permutationally invariant, which allows for its reconstruction using polynomial resources [28, 29]. Turning from states to processes, a structural feature of channels may be that they are given by a network of gates. Under suitable assumptions on the figure of merit, it turns out that independent measurements on the components are optimal [30]. This illustrates the more general challenge of building estimation schemes that utilize structure and symmetry of the underlying system—be it physically or operationally motivated. We believe that there is both the potential and the urgent need to explore this line of research further.

New impetus for quantum tomography developments has repeatedly come from novel non-trivial developments in classical machine-learning theory. Indeed, the problem of turning huge and noisy data sets into meaningful information is by no means unique to the quantum laboratory. In the classical world, the ubiquity of the internet and the availability of cheap sensors in areas as diverse as life sciences and industrial applications has given rise to the paradigm of big data. We have recently seen several instances—compressed sensing being a prominent example—where ideas have flowed in both directions between researchers working on classical high-dimensional data analysis on the one hand, and quantum physicists thinking about new theoretical models for tomography on the other.

We would also like to draw attention to the fact that new directions in tomography have recently been driven by the unique needs of quantum cryptography. Here, the need for absolutely rigorous statements concerning the uncertainties of the available resources is particularly acute. This has led to a new perspective on the concept of region estimators for quantum problems and to the revisiting of intrinsic challenges such as incomplete measurements, imperfectly characterized detectors, finite data and other error sources [31–41]. This development also serves as a reminder that generic, off-the-shelf methods are often insufficient for highly specialized applications.

The editors agree that the appeal of the subject lies largely in the breadth of techniques that must be brought together in order to fully understand the problem. To live up to the highest standards, it is essential to have a thorough understanding of the particular experiment generating the data; one needs a solid grasp of theoretical physics to understand the uniquely quantum mechanical aspects; a rigorous error analysis requires knowledge of mathematical statistics; and lastly, non-trivial problems in numerical analysis need to be solved. We believe that more communication between researchers working in these very different fields is crucial for further progress. It is our hope that this 'focus on' collection provides a platform for facilitating this necessary exchange of ideas.

References

[1] Vogel K and Risken H 1989 Determination of quasiprobability distributions in terms of probability distributions for the rotated quadrature phase *Phys. Rev.* A **40** 2847–9

[2] Smithey D T, Beck M, Raymer M G and Faridani A 1993 Measurement of the wigner distribution and the density matrix of a light mode using optical homodyne tomography: application to squeezed states and the vacuum *Phys. Rev. Lett.* **70** 1244–7

[3] Laiho K, Avenhaus M and Silberhorn C 2012 Characteristics of displaced single photons attained via higher order factorial moments *New J. Phys.* **14** 105011

[4] Brańczyk A M, Mahler D H, Rozema L A, Darabi A, Steinberg A M and James D F V 2012 Self-calibrating quantum state tomography *New J. Phys.* **14** 085003

[5] Müller C R, Stoklasa B, Peuntinger C, Gabriel C, Řeháček J, Hradil Z, Klimov A B, Leuchs G, Marquardt Ch and Sánchez-Soto L L 2012 Quantum polarization tomography of bright squeezed light *New J. Phys.* **14** 085002

[6] Chiuri A, Mazzola L, Paternostro M and Mataloni P 2012 Tomographic characterization of correlations in a photonic tripartite state *New J. Phys.* **14** 085006

[7] Wallentowitz S, Seifert B and Godoy S 2012 Local sampling of the quantum phase-space distribution of a continuous-wave optical beam *New J. Phys.* **14** 105019

[8] Bogdanov Yu I, Kalinkin A A, Kulik S P, Moreva E V and Shershulin V A 2013 Quantum polarization transformations in anisotropic dispersive media *New J. Phys.* **15** 035012

[9] Söderholm J, Björk G, Klimov A B, Sánchez-Soto L L and Leuchs G 2012 Quantum polarization characterization and tomography *New J. Phys.* **14** 115014

[10] Sayrin C, Dotsenko I, Gleyzes S, Brune M, Raimond J M and Haroche S 2012 Optimal time-resolved photon number distribution reconstruction of a cavity field by maximum likelihood *New J. Phys.* **14** 115007

[11] Christensen S L, Béguin J B, Sørensen H L, Bookjans E, Oblak D, Müller J H, Appel J and Polzik E S 2013 Toward quantum state tomography of a single polariton state of an atomic ensemble *New J. Phys.* **15** 015002

[12] Rey-de Castro R, Cabrera R, Bondar D I and Rabitz H 2013 Time-resolved quantum process tomography using Hamiltonian-encoding and observable-decoding *New J. Phys.* **15** 025032

[13] Mitchell M W, Koschorreck M, Kubasik M, Napolitano M and Sewell R J 2012 Certified quantum non-demolition measurement of material systems *New J. Phys.* **14** 085021

[14] Guţă M, Kypraios T and Dryden I 2012 Rank-based model selection for multiple ions quantum tomography *New J. Phys.* **14** 105002

[15] Zhang L, Datta A, Coldenstrodt-Ronge H B, Jin X-M, Eisert J, Plenio M B and Walmsley I A 2012 Recursive quantum detector tomography *New J. Phys.* **14** 115005

[16] Brida *et al* 2012 Quantum characterization of superconducting photon counters *New J. Phys.* **14** 085001

[17] Anis A and Lvovsky A I 2012 Maximum-likelihood coherent-state quantum process tomography *New J. Phys.* **14** 105021

[18] Häffner H, Hänsel W, Roos C F and Benhelm J 2005 Scalable multiparticle entanglement of trapped ions *Nature* **438** 643–6

[19] Gross D, Liu Y-K, Flammia S T, Becker S and Eisert J 2010 Quantum state tomography via compressed sensing *Phys. Rev. Lett.* **105** 150401

[20] Gross D 2011 Recovering low-rank matrices from few coefficients in any basis *IEEE Trans. Inform. Theory* **57** 1548–66

[21] Flammia S T, Gross D, Liu Y-K and Eisert J 2012 Quantum tomography via compressed sensing: error bounds, sample complexity and efficient estimators *New J. Phys.* **14** 095022

[22] Ohliger M, Nesme V and Eisert J 2013 Efficient and feasible state tomography of quantum many-body systems *New J. Phys.* **15** 015024

[23] Hastings M B 2006 Solving gapped Hamiltonians locally *Phys. Rev.* B **73** 085115

[24] Cramer M, Plenio M B, Flammia S T, Somma R, Gross D, Bartlett S D, Landon-Cardinal O, Poulin D and Liu Y-K 2010 Efficient quantum state tomography *Nature Commun.* **1** 149

[25] Landon-Cardinal O and Poulin D 2012 Practical learning method for multi-scale entangled states *New J. Phys.* **14** 085004

[26] Baumgratz T, Gross D, Cramer M and Plenio M B 2013 Scalable reconstruction of density matrices *Phys. Rev. Lett.* **111** 020401

[27] Baumgratz T, Nüßeler A, Cramer M and Plenio M B 2013 A scalable maximum likelihood method for quantum state tomography *New J. Phys.* **15** 125004

[28] Toth G, Wieczorek W, Gross D, Krischek R, Schwemmer C and Weinfurter H 2010 Permutationally invariant quantum tomography *Phys. Rev. Lett.* **105** 250403

[29] Moroder T, Hyllus P, Tóth G, Schwemmer C, Niggebaum A, Gaile S, Gühne O and Weinfurter H 2012 Permutationally invariant state reconstruction *New J. Phys.* **14** 105001

[30] Chiribella G 2012 Optimal networks for quantum metrology: semidefinite programs and product rules *New J. Phys.* **14** 125008

[31] van Enk S J and Blume-Kohout R 2013 When quantum tomography goes wrong: drift of quantum sources and other errors *New J. Phys.* **15** 025024

[32] Teo Y S, Englert B-G, Řeháček J, Hradil Z and Mogilevtsev D 2012 Verification of state and entanglement with incomplete tomography *New J. Phys.* **14** 105020

[33] Sugiyama T, Turner P S and Murao M 2012 Effect of non-negativity on estimation errors in one-qubit state tomography with finite data *New J. Phys.* **14** 085005

[34] Langford N K 2013 Errors in quantum tomography: diagnosing systematic versus statistical errors *New J. Phys.* **15** 035003

[35] Ng H K, Phuah K T B and Englert B-G 2012 Minimax mean estimator for the trine *New J. Phys.* **14** 085007

[36] Gendra B, Ronco-Bonvehi E, Calsamiglia J, Muñoz-Tapia R and Bagan E 2012 Beating noise with abstention in state estimation *New J. Phys.* **14** 105015

[37] Glancy S, Knill E and Girard M 2012 Gradient-based stopping rules for maximum-likelihood quantum-state tomography *New J. Phys.* **14** 095017

[38] Christandl M and Renner R 2012 Reliable quantum state tomography *Phys. Rev. Lett.* **109** 120403

[39] Blume-Kohout R 2010 Optimal, reliable estimation of quantum states *New J. Phys.* **12** 043034

[40] Mogilevtsev D, Řeháček J and Hradil Z 2012 Self-calibration for self-consistent tomography *New J. Phys.* **14** 095001

[41] Mogilevtsev D, Ignatenko A, Maloshtan A, Stoklasa B, Rehacek J and Hradil Z 2013 Data pattern tomography: reconstruction with an unknown apparatus *New J. Phys.* **15** 025038