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Quantum formulation of the Einstein equivalence principle

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The validity of just a few physical conditions comprising the Einstein equivalence principle (EEP) suffices to ensure that gravity can be understood as spacetime geometry. The EEP is therefore subject to ongoing experimental verification, with present-day tests reaching the regime in which quantum mechanics becomes relevant. Here we show that the classical expression of the EEP does not apply in such a regime. The EEP requires equivalence between the rest mass-energy of a system, the mass-energy that constitutes its inertia, and the mass-energy that constitutes its weight. In quantum mechanics, the energy contributing to the mass is given by a Hamiltonian operator of the internal degrees of freedom. Therefore, we introduce a quantum expression of the EEP—equivalence between the rest, inertial and gravitational internal energy operators. Validity of the classical EEP does not imply the validity of its quantum formulation, which thus requires independent experimental verification. We propose new tests as well as re-analysing existing experiments, and we discuss to what extent they allow quantum aspects of the EEP to be tested.

General relativity describes a particular field among other fields in nature: the field dynamics is governed by the distribution of matter, while the dynamics of matter is universally governed by the field. The former feature is why general relativity can be seen as a theory of gravity, the latter allows us to interpret the field as the spacetime metric. For this metric interpretation of gravity to be possible, all interactions must satisfy certain conditions. They follow from the hypothesis first formulated by Einstein[1], stating the equivalence with respect to all laws of physics between a coordinate system in uniform acceleration and a stationary coordinate system in a homogeneous gravitational field. Applying the equivalence to non-relativistic physics yields universality of the gravitational acceleration of free-fall (an empirical fact known since at least the sixth century[2]) while still keeping the picture of gravity as a force. Applying the equivalence to all physical laws implies that gravitational and fictitious forces cannot be distinguished even in principle. Free-fall can therefore be seen as an inertial motion—along a 'straight line', but in spacetime that is generically not flat. In particular, if special relativity holds, the equivalence hypothesis entails that spacetime can be described as a Lorentzian manifold.

The EEP comprises validity of the equivalence hypothesis and of special relativity. Its fundamental importance comes from the fact that it provides conditions that physical interactions must satisfy in order that gravity can be interpreted as curved spacetime geometry, but the principle is formulated independently of the mathematical framework of general relativity. The role of the EEP for physical theories is to constrain the allowed form of dynamics: for the validity of special relativity, internal energy must contribute equally to the rest mass and to inertia; for homogeneous gravity to be equivalent to acceleration, the internal energy must contribute equally to the rest mass and to the weight; for free-fall to be universal, the inertia and weight must be equal. These three conditions express (at low energies) the principles of local Lorentz invariance (LLI), local position invariance (LPI) and the weak equivalence principle (WEP), respectively, which comprise the EEP[3]. Validity of the EEP requires that the mass-energy of a system is a universal quantity. (The system's action is then described by the proper length of its world line on a spacetime manifold, and coordinates established by physical 'rods and clocks' yield a spacetime manifold that is Lorentzian.) For the above reasons, validity of the EEP is tested by measuring inertial and gravitational masses and contributions of the binding energies to the mass, for systems with different internal composition[3].

Here we analyse the EEP in quantum mechanics. In quantum formalism, internal energy is described by a Hermitian operator, governing dynamics of the internal degrees of freedom (DOFs). Present-day tests of the EEP, however, are sensitive only to violations that would alter the spectra of internal energies. Such violations can be expressed in terms of differences between diagonal elements of the operators. However, to test the validity of the EEP in quantum mechanics, it is necessary to verify the equivalence also for the off-diagonal elements. Effectively, it has been assumed that internal energy operators must commute, and EEP violations can only alter their eigenvalues. Here we introduce a quantum formulation of the EEP and a corresponding test theory (applicable to low-energy, laboratory experiments) that lift this assumption. Our approach shows that testing the EEP in quantum mechanics differs both conceptually and quantitatively from testing it in classical physics. Our results open new experimental possibilities, including tests of the quantum aspects of the EEP. Testing genuine quantum features of the EEP, with no classical analogues, is feasible with current technology: the first experiment has already been realized[4], establishing a bound of the order of 10^{-8} on the violations of the quantum formulation of the WEP.

Massive particles with quantized internal energy

Analysis of the EEP in quantum theory requires a framework of relativistic quantum particles with internal DOFs. For the case when the EEP holds, such a framework is developed in refs [5–7]. To lowest post-Newtonian order, the Hamiltonian describing a low-energy composite particle with internal energy $\hat{H}_{\text{int}}$, position and momentum $\hat{Q}$ and $\hat{P}$, and mass m reads

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$$\hat{H} = mc^2 + \frac{\hat{P}^2}{2m} + m\phi(\hat{Q}) + \hat{H}_{\mathrm{int}} - \hat{H}_{\mathrm{int}}\frac{\hat{P}^2}{2m^2c^2} + \hat{H}_{\mathrm{int}}\frac{\phi(\hat{Q})}{c^2} \quad (1)$$

where $\phi(x)$ denotes the gravitational potential (see Supplementary Note 1 for derivation).

$\hat{H}$ applies in the regime in which the centre of mass (CM) of a particle is effectively non-relativistic, but its internal evolution is fast and thus sensitive to relativistic corrections. It applies, for example, to atoms or molecules in laboratory experiments. The first correction, $\hat{H}_{\mathrm{int}}\left(\hat{I}_{\mathrm{int}} - \frac{\hat{P}^2}{2m^2c^2}\right)$, describes special relativistic time dilation of the internal dynamics; the second, $\hat{H}_{\mathrm{int}}\left(\hat{I}_{\mathrm{int}} + \frac{\phi(\hat{Q})}{c^2}\right)$, describes its gravitational time dilation. When the CM is effectively classical, these corrections are routinely tested with atomic clocks[8,9]. In the regime in which both internal and external DOFs require a quantum description, these terms lead to new effects in interference experiments with clocks[5,10–12] and result in an effective decoherence mechanism for sufficiently complex systems[6,13].

The above time dilation terms can directly be obtained by extending the mass–energy equivalence[14] to quantum theory. (The reasoning is analogous to the classical case[15].) According to the mass–energy equivalence, changing a body's internal energy by E changes its mass: $m \to m + E/c^2$ (experimentally verified[16] to 4×10^{-7}). This holds for any internal state, and owing to the linear structure of the state-space of quantum theory, the equivalence must hold also for any superposition of internal states. The total mass-energy of a composite quantum system can therefore be described by a mass-energy operator

$$\hat{M} = m\hat{I}_{\mathrm{int}} + \frac{\hat{H}_{\mathrm{int}}}{c^2} \quad (2)$$

The rest-mass parameter m can be operationally defined as the static part (ground state) of the total mass-energy, whereas $\hat{H}_{\mathrm{int}}$ is its dynamical part (driving non-trivial internal evolution). One immediately sees that equation (1) can be obtained by incorporating quantum mass–energy equivalence into a non-relativistic Hamiltonian $mc^2 + \frac{\hat{P}^2}{2m} + m\phi(\hat{Q})$ by taking $m \to \hat{M}$ and keeping terms to lowest order in $\hat{H}_{\mathrm{int}}/mc^2$.

The model

Based on the standard quantum theory, we now construct a phenomenological test theory for analysing the EEP in quantum mechanics. Standard approaches[3] introduce different inertial and gravitational masses m_{i} and m_{g} and the corresponding different (inertial and gravitational) internal energy energy values for internal eigenstates. We generalize this approach and consider that the entire mass-energy operators could be different when describing the gravitational $\hat{M}_{\mathrm{g}}$, inertial $\hat{M}_{\mathrm{i}}$ and rest mass-energy $\hat{M}_{\mathrm{r}}$. In analogy to equation (2), these operators have a static (mass) and a dynamical contribution:

$$\hat{M}_{\alpha} := m_{\alpha}\hat{I}_{\mathrm{int}} + \frac{\hat{H}_{\mathrm{int},\alpha}}{c^2} \quad (3)$$

where α = r, i, g; $\hat{H}_{\mathrm{int,i}}$ and $\hat{H}_{\mathrm{int,g}}$ are the internal energies contributing to m_{i} and m_{g}, respectively; and $\hat{H}_{\mathrm{int,r}}$ is the internal energy contributing to the rest mass m_{r}. In the present context $\hat{H}_{\mathrm{int,r}}$ can be operationally defined as the total energy of the particle at rest far from massive objects (and m_{r} has the physical meaning of an active gravitational mass, but because the gravitational field generated by the particle is not considered here, it can be assigned an arbitrary value). With the mass-energies defined in equation (3), we obtain, using equation (1), our test theory

$$\hat{H}^{\mathrm{Q}}_{\mathrm{test}} = m_{\mathrm{r}}c^2 + \hat{H}_{\mathrm{int,r}} + \frac{\hat{P}^2}{2m_{\mathrm{i}}} + m_{\mathrm{g}}\phi(\hat{Q}) - \hat{H}_{\mathrm{int,i}}\frac{\hat{P}^2}{2m_{\mathrm{i}}^2c^2} + \hat{H}_{\mathrm{int,g}}\frac{\phi(\hat{Q})}{c^2} \quad (4)$$

The Hamiltonian $\hat{H}^{\mathrm{Q}}_{\mathrm{test}}$ is a new test theory for analysing the EEP: it applies in the regime in which the relevant (internal) DOFs of test-bodies are quantized, and not simply discretized. The validity of the WEP is here expressed by $\hat{M}_{\mathrm{i}} = \hat{M}_{\mathrm{g}}$ (universality of free-fall); of LLI by $\hat{H}_{\mathrm{int,r}} = \hat{H}_{\mathrm{int,i}}$ (universality of special relativistic time dilation); and of LPI by $\hat{H}_{\mathrm{int,r}} = \hat{H}_{\mathrm{int,g}}$ (universality of the gravitational time dilation). Validity of the EEP in quantum theory (at low energies) therefore requires

$$\hat{M}_{\mathrm{r}} = \hat{M}_{\mathrm{i}} = \hat{M}_{\mathrm{g}} \quad (5)$$

Conditions (5) entail equivalence of the diagonal as well as the off-diagonal elements of the internal energy operators—their eigenvalues as well as eigenbases. Testing the validity of the EEP for an n-level quantum system necessitates comparing elements of Hermitian operators $\hat{M}_{\alpha}$ and thus measuring $2n^2 - 1$ (real) parameters (parameter m_{r} is arbitrary); see Table 1. Conditions (5) and equation (4) can equivalently be derived by requiring validity of Einstein's hypothesis of equivalence for a low-energy composite quantum particle[7].

If internal energies are incorporated classically, the conditions for validity of the EEP are a special case of equation (5) (see Table 1). The corresponding semi-classical test theory reads

$$H^{\mathrm{C}}_{\mathrm{test}} = m_{\mathrm{r}}c^2 + E_{\mathrm{r}} + \frac{\hat{P}^2}{2m_{\mathrm{i}}} + m_{\mathrm{g}}\phi(\hat{Q}) - E_{\mathrm{i}}\frac{\hat{P}^2}{2m_{\mathrm{i}}c^2} + E_{\mathrm{g}}\frac{\phi(\hat{Q})}{c^2} \quad (6)$$

where $M_{\alpha} := m_{\alpha} + E_{\alpha}/c^2$ are values of the different mass-energies. Here, validity of the EEP requires $M_{\mathrm{r}} = M_{\mathrm{i}} = M_{\mathrm{g}}$ for each internal state and entails probing $2n - 1$ parameters for a system with n internal states. The conditions and the number of parameters to test are the same in $H^{\mathrm{C}}_{\mathrm{test}}$ and in a fully classical theory, as they are independent of whether the external DOFs require a quantum description. Importantly, equation (5) reduces to the classical case if operators $\hat{H}_{\mathrm{int},\alpha}$ commute and thus only differ only in their spectra. The classical conditions for validity of the EEP can be obtained by restricting the quantum conditions (5) to the diagonal elements.

Consequences for 'quantum tests' of the EEP

At present, a distinction is made between classical and quantum tests of the EEP[17–20] based on the experimental method used. However, verifying the validity of the EEP in classical and in quantum theory requires testing different concepts (equality of the eigenvalues and commutativity of the mass-energy operators in quantum theory, versus equality of the eigenvalues (or, equivalently, diagonal elements) in the classical limit). We thus propose to distinguish between tests of the classical and of the quantum formulation of the EEP. Tests of the classical formulation comprise experiments

Table 1 | Einstein equivalence principle in classical and in quantum theory

Physical regime		EEP WEP	LLI	LPI	Number of parameters
Newtonian	Classical and quantum	$m_i = m_g$	–	–	1
Newtonian and mass–energy equivalence	Classical	$m_i c^2 + E_i = m_g c^2 + E_g$	$E_r = E_i$	$E_r = E_g$	$2n-1$
	Quantum	$m_i c^2 \hat{I} + \hat{H}_i = m_g c^2 \hat{I} + \hat{H}_g$	$\hat{H}_r = \hat{H}_i$	$\hat{H}_r = \hat{H}_g$	$2n^2-1$

Validity conditions and the number of parameters to test in an n-level system (up to $1/c^2$). The Newtonian limit of the WEP, that is the equivalence of the inertial m_i and gravitational m_g mass parameters suffices to ensure validity of the EEP in the non-relativistic classical and quantum physics. Beyond that limit, the EEP also requires LLI and LPI, which physically mean universality of special and general relativistic time dilation, respectively. In quantum physics, LLI and LPI hold if the rest, inertial and gravitational internal energy operators $\hat{H}_\alpha$, $\alpha = $r,i,g, are equal, whereas in classical physics equality of the internal energy values E_α suffices. Therefore, validity of the EEP in classical theory does not guarantee its validity in quantum theory beyond the non-relativistic regime. Reprinted from ref. [7], Springer.

whose non-null results (indicating a violation) can be explained by a 'diagonal' test theory in which internal energy operators commute, like in H^{C}_{test}. Experiments for which the explanation of the results requires non-commuting mass-energy operators, as in $\hat{H}^{Q}_{test}$, qualify as tests of the quantum formulation of the EEP. Importantly, it is not sufficient to make an EEP test with 'a quantum system' to probe the quantum formulation of the principle (just as it is not sufficient to test correlations between 'quantum systems' to distinguish between classical correlations and entanglement).

Table 2 summarizes how the violations of the quantum formulation of the EEP result in distinct physical effects, not present when its classical formulation is violated. These effects include generation of entanglement between external and internal DOFs in free-fall (Fig. 1), anomalous state transitions (Fig. 2), modulation of the interference contrast (Fig. 3) and excess phase noise[4]. See Methods for details of the experimental set-ups discussed in the figures and for estimates of the bounds on quantum-EEP violations.

Comparison to other theoretical approaches

There are two sides of the theoretical analysis of the EEP. One is a phenomenological (or conceptual) analysis: how can the violations of the EEP most generally be manifested in quantum physics, and how can we test them? The second is a formulation of concrete theories violating the EEP which can provide 'fine-graining' of the phenomenological conditions in terms of new physics (such as new fields in the Standard Model[23], multiple spacetime metrics[24] and so on). Such fine-grained models compared with experiments constrain the possible EEP violations as well as the underlying new physics. Current phenomenological approaches assume that the most general EEP violations can be expressed in terms of the difference in the values of the total mass-energies. For example, ref. [3] considers the most general WEP violation to be expressed by $M_g = M_i + \sum_A \eta^A \frac{E^A}{c^2}$, where A labels different interactions, E^A is their corresponding energy, and η^A parameterizes potential WEP violations. Here WEP holds if $\eta^A = 0$. Dynamical frameworks providing fine-graining of these conditions include $TH\epsilon\mu$-formalism[25], Hamiltonians with (spatial) mass-tensors[26], a modified Pauli equation (predicting spin-coupled masses for elementary particles and fields)[27] and the Standard Model extensions (SME)[23]—the Standard Model of particle physics including a large class of terms that violate the EEP (and charge, parity and time-reversal, CPT). Also, field theories in multiple spacetime metrics have been formulated and discussed in the context of EEP violations[24].

Our work extends the very phenomenological approach by expressing the validity of the EEP as an equation for the mass-energy operators; equation (5). This allows us to describe the possibility that EEP violations lead to eigenstates of, say, weight $|E^{int}_{g\alpha}\rangle$ not being eigenstates of inertia $|E^{int}_{i\alpha}\rangle$, that is, $|E^{int}_{g\alpha}\rangle \propto |E^{int}_{i\alpha}\rangle + \sum_\beta \eta_{\alpha\beta} |E^{int}_{i\beta}\rangle$ (here the WEP holds if the entire matrix $\eta_{\alpha\beta}$ is identically 0). In other words, a system in an eigenstate of weight—with a well-defined gravitational mass-energy value M_g—could be in a superposition of different inertial mass-energies if the EEP is violated. This cannot be expressed by considering merely different values of the inertial and gravitational mass-energies.

The test theory $\hat{H}^{Q}_{test}$ provides a minimal dynamical framework for analysing experiments in which the dynamics of the quantized, and not just discretized, internal DOFs is relevant. One could ask whether some of the existing frameworks can provide fine-graining of the quantum formulation of the EEP or allow $\hat{H}^{Q}_{test}$ to be derived. The frameworks of refs [25,26] contain only mass-energy parameters for the different energy levels (they are of the general form (6)) and are thus strictly less general than $\hat{H}^{Q}_{test}$. The modified Pauli equation was shown[27] to have the same number of parameters as its classical limit (spin effects were shown to be in principle measurable using polarized bulk matter), unlike $\hat{H}^{Q}_{test}$. Our model is also more general than the low-energy limit of the multimetric theories[24] (see Supplementary Note 2). The SME framework appears promising to provide $\hat{H}^{Q}_{test}$ as a low-energy approximation, but this remains an open question. For analysing low-energy bound systems, such as atoms, the effective models derived from the SME thus far have all assumed from the beginning that the EEP-violating parameters can only modify the values of the internal/binding energies[28,29]. The off-diagonal elements (commutativity) of the internal energies have not been considered. However, because SME modifies the fundamental interactions between elementary particles, the eigenvalues as well as the eigenstates of the resulting bound systems in general will be affected. Interestingly, certain combinations of the SME parameters have never appeared in experiments analysed thus far[30] and remain unconstrained by the available data. We conjecture that at least some of the untested combinations appear in the off-diagonal elements of the internal energy operators and will become accessible in tests of the quantum formulation of the EEP. For a direct comparison of our approach and action-based frameworks, the Lagrangian corresponding to $\hat{H}^{Q}_{test}$ is derived in Supplementary Note 3. We stress, however, that a phenomenological approach is complementary to the formulation of specific theories predicting EEP violations, as it tests the entire framework and provides restrictions on the possible EEP-violating future theories.

EEP-violating mass operators have thus far been considered only in the context of the WEP for neutrinos: refs [31,32] studied a model in which neutrinos have equal masses and their flavour oscillations occur because of a flavour-nondiagonal coupling to gravity, violating the equivalence principle. Such effects are excluded up to 10^{-11} (ref. [31]) for massless neutrinos and ruled out for the massive electronic–muonic neutrinos[32]. Moreover, in the neutrino sector of the SME, various modifications of the Pontecorvo–Maki–Nakagawa–Sakata matrix were studied (that is, modified neutrino mass-values and mixing angles)[33]. Note that in a reference frame in which the neutrino's CM energy is

Table 2 | Tests of the classical and of the quantum formulation of the EEP

Experiment	Effects of classical-EEP violation	Effects of quantum-EEP violation
	(compatible with $[\hat{M}_\alpha, \hat{M}_\beta] = 0$)	(incompatible with $[\hat{M}_\alpha, \hat{M}_\beta] = 0$)
Atom interferometry	**Anomalous phase shift**	**Visibility modulations** (Fig. 3)
Atoms in an internal energy eigenstate	Explained by: diag$\hat{M}_g \neq$ diag$\hat{M}_i$	Require: $[\hat{M}_g, \hat{M}_i] \neq 0$
	Tested, for example, in refs [4,18,21]	No direct test; see Methods
Atom interferometry	**As above**	**Excess phase noise** (for random relative phase of the superposition)
Atoms in a superposition of internal energy eigenstates		Require: $[\hat{M}_g, \hat{M}_i] \neq 0$
		First tested in ref. [4]
Frequency comparison Spectroscopy	**Anomalous frequency/line shift**	**Anomalous state transition** (Fig. 2)
	Explained by: diag$\hat{H}_r \neq$ diag$\hat{H}_{i(g)}$	Require: $[\hat{H}_r, \hat{H}_{i(g)}] \neq 0$
	Tested, for example, in refs [8,9]	No direct test; see Methods
Free-fall, for example, of antihydrogen ($\bar{H}$)	**Different free-fall times** (for instance, of $\bar{H}$ as compared with hydrogen)	**Entanglement between internal state and position** (Fig. 1)
	Explained by: diag$\hat{M}_g \neq$ diag$\hat{M}_i$	Require: $[\hat{M}_i, \hat{M}_g] \neq 0$
	Preliminary test in ref. [22]	No direct test; see Methods
		Internal state oscillations
		Require: $[\hat{M}_i, \hat{M}_r] \neq 0$
		No direct test; see Methods

Low-energy experiments testing the EEP with massive systems and physical effects that would arise therein from violations of the classical and of the quantum formulation of the principle. The effects detectable in tests realized or proposed thus far (except that in ref. [4]) allow one to probe only the classical formulation of the EEP: the experiments were designed to test equivalence of the inertial and gravitational masses, or to test the equivalence of the energy spectra, or to measure the mean accelerations of free-fall. Any EEP violation in such tests could be explained by a modification of only the diagonal elements of the mass-energy operators.

comparable to its rest energy (consistent for massive neutrinos), the above approach would generally yield a low-energy Hamiltonian of the form $\hat{H}^Q_{test}$—where the rest, inertial and gravitational mass-energies are described by Hermitian operators and not by classical parameters. This corroborates our finding that incorporating mass-energies as operators is necessary for a complete analysis of the EEP for composite quantum particles. Finally, we note a phenomenological approach for studying WEP violations for unstable particles, using a semi-classical treatment of the decay via an imaginary mass parameter[34].

Discussion

Because testing LPI is much more challenging than for the WEP—which is the best constrained part of the EEP—it is of practical importance to understand under what assumptions tests of the WEP and LPI become related. The following assumptions are incorporated in our framework: (1) energy is conserved; (2) in the non-relativistic limit, and for equal inertial and gravitational masses, standard quantum theory is recovered; (3) mass-energy equivalence holds in quantum mechanics as per equation (3). Under these assumptions, no single principle within the EEP implies the others. If an additional assumption is made—(4) that LLI is valid—then bounds on the WEP can also constrain LPI violations. This holds in the classical as well as in the quantum case. All four assumptions are also made in the well-known thought experiment of Nordtvedt[35], often presented as an argument that energy conservation alone

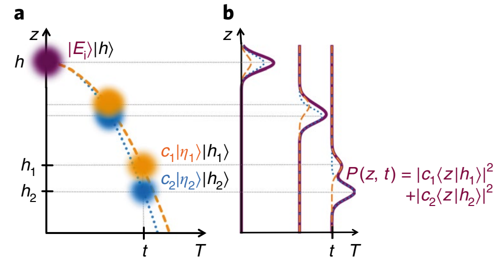

Fig. 1 | Testing quantum formulation of the WEP. Evolution of the CM of a quantum system free-falling in a gravitational field g if the WEP is violated. The system is prepared in a product state of the internal energy $|E_i\rangle$ and the CM position $|h\rangle$ (described by a Gaussian distribution centred at h). If the quantum formulation of the WEP is violated, the system falls with different accelerations in superposition. **a**, Semi-classical trajectories of the CM (dashed orange and dotted blue lines) become correlated with internal states $|\eta_1\rangle$ and $|\eta_2\rangle$, for which the free-fall acceleration is well-defined and correspondingly reads $\eta_1 g$ and $\eta_2 g$. **b**, Probability of finding the system around z after free-fall time t (see Methods); purple line. The probabilities conditioned on the internal states $|\eta_1\rangle$ and $|\eta_2\rangle$ are marked with dashed orange and dotted blue lines, respectively. Assuming linearity of quantum theory, modulations in $P(z,t)$ would probe violations of the quantum formulation of the WEP. Testing entanglement between internal and CM states generated in this scenario would furnish a test of the quantum formulation of the WEP without this additional assumption. Reproduced from ref. [7], Springer.

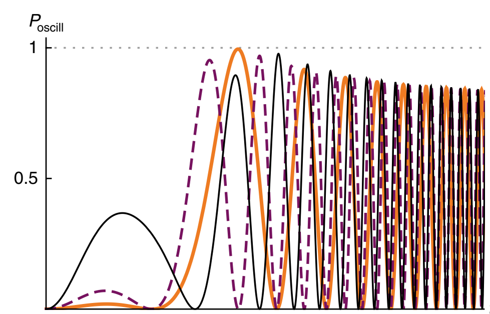

Fig. 2 | Internal state oscillations for testing quantum formulation of LLI and LPI. Probability of an internal state transition induced by violations of the quantum EEP for a free-falling particle in a homogeneous gravitational field: a qualitative picture. The figure assumes that the WEP holds but LLI and LPI can be violated. The black and the dashed purple lines illustrate the effect for two different initial heights of free-fall, which is greater for the black-line plot. The thick orange plot corresponds to the same height as the dashed purple plot, but for a particle with half the mass. For all plots, $|\alpha_q| \approx 10\%$, where α_q parameterizes violations of the quantum EEP (see Methods). Under violations of the classical formulation (when internal energy operators commute), the oscillation probability in this scenario is zero. Such an experiment would thus directly probe quantum formulation of the EEP.

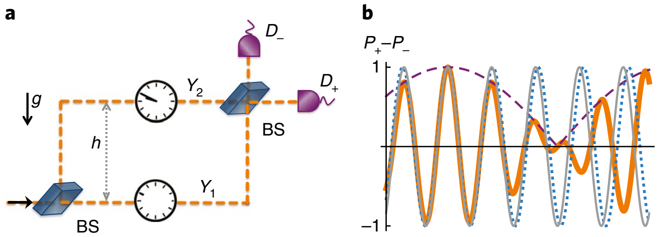

Fig. 3 | Interferometric test of the quantum formulation of LPI. Mach–Zehnder interferometer for testing quantum formulation of LPI and qualitative data for different scenarios. **a**, The interferometer uses a pair of beam splitters (BS) and detectors $D_{\pm}$, and is stationary in the laboratory reference frame with gravitational acceleration g. The two trajectories γ_1, γ_2 allowed by the set-up are vertically separated by h. Initial internal state is assumed to be stationary along γ_1. **b**, Detection probabilities in different scenarios: if LPI holds, there will be a gravitational phase shift of the interference pattern (thin grey line) with ideally unit visibility. Measuring a different phase shift (dotted blue line) would mean a violation of the classical LPI (it can be explained by a diagonal test theory). Changes in the interference visibility (dashed purple line) for the initial state chosen here can only arise from the non-commutativity of the internal energy operators. Probabilities of detection in such a case are represented by the thick orange line. The internal state would here be stationary when following γ_1 but not for γ_2. Therefore the particle is represented as a ticking 'clock' only on γ_1. The visibility of gravitationally induced interference, with the internal state chosen here, is sensitive to the quantum violations of LPI, whereas the phase shift is sensitive only to the violations of the classical LPI. Reproduced from ref. [7], Springer.

implies that tests of the WEP and of LPI are equivalent. We defer further discussion of controversies on the status of the EEP in quantum theory to the Methods.

Methods

Methods, including statements of data availability and any associated accession codes and references, are available at https://doi.org/10.1038/s41567-018-0197-6.

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Methods

Similar to other parameterized frameworks[23,25–27,33], our test theory does not predict EEP violations but provides quantitative bounds on the EEP-violating effects when compared with experimental data. Deriving quantitative benchmarks for the violations from complete EEP-violating theories remains an important topic for future work.

To obtain numerical bounds on quantum-EEP violations, we first introduce their parameterization: $\hat{\eta} := \hat{I}_{\rm int} - \hat{M}_{\rm g}\hat{M}_{\rm i}^{-1}$ for the violations of the WEP; $\hat{\beta} := \hat{I}_{\rm int} - \hat{H}_{\rm int,i}\hat{H}_{\rm int,r}^{-1}$ for LLI; $\hat{\alpha} := \hat{I}_{\rm int} - \hat{H}_{\rm int,g}\hat{H}_{\rm int,r}^{-1}$ for LPI. Violations of the classical formulation of the EEP are described by a real parameter $\eta_{\rm class} := 1 - M_{\rm g}/M_{\rm i}$ per internal state for the WEP; parameter $\beta_{\rm class} := 1 - E_{\rm i}/E_{\rm r}$ and $\alpha_{\rm class} := 1 - E_{\rm g}/E_{\rm r}$, respectively, for LLI and LPI per internal state (apart from the ground state). Classical parameters $\eta_{\rm class}$, $\alpha_{\rm class}$ and $\beta_{\rm class}$ can be interpreted as the diagonal elements of the quantum parameter-matrices $\hat{\eta}$, $\hat{\alpha}$ and $\hat{\beta}$.

Testing the quantum formulation of the WEP. In classical theory, the WEP entails universal acceleration in free-fall. In quantum theory, this means that the time evolution of the CM position operator is independent of the internal DOFs.

In the Heisenberg picture, time evolution of an observable $\hat{A}$ under a Hamiltonian $\hat{H}$ reads $d\hat{A}/dt = -i/\hbar[\hat{A}, \hat{H}]$. We are in fact interested in the CM 'acceleration' operator $\hat{a}_{\hat{H}^{\rm Q}_{\rm test}} := d^2\hat{Q}/dt^2$, which reads

$$\hat{a}_{\hat{H}^{\rm Q}_{\rm test}} = -\hat{M}_{\rm g}\hat{M}_{\rm i}^{-1}\nabla\phi(\hat{Q}) + \frac{i}{\hbar}[\hat{H}_{\rm int,i}, \hat{H}_{\rm int,r}]\frac{\hat{P}}{m_{\rm i}c^2} + \mathcal{O}(1/c^4) \tag{7}$$

The commutator in equation (7) expresses a violation of LLI as it is independent of the gravitational potential. In a test theory $H^{\rm C}_{\rm test}$, the CM acceleration $\hat{a}_{H^{\rm C}_{\rm test}}$ reads

$$\hat{a}_{H^{\rm C}_{\rm test}} = -M_{\rm g}M_{\rm i}^{-1}\nabla\phi(\hat{Q}) \tag{8}$$

and indeed is a special case of equation (7) for $[\hat{H}_{\rm int,i}, \hat{H}_{\rm int,r}] = 0$.

Classical WEP violations (when $\hat{M}_{\rm i} \neq \hat{M}_{\rm g}$ but $[\hat{H}_{\rm int,i}, \hat{H}_{\rm int,g}] = 0$) would lead to different free-fall accelerations for different internal energy eigenstates. Violations of the quantum WEP (when $[\hat{H}_{\rm int,i}, \hat{H}_{\rm int,g}] \neq 0$) lead to further effects. For simplicity, below, we assume LLI. Because of equation (7) and $\hat{M}_{\rm g}\hat{M}_{\rm i}^{-1} = \hat{I}_{\rm int} - \hat{\eta}$, only eigenstates of $\hat{\eta} \approx m_{\rm g}/m_{\rm i}(\hat{I} + \hat{H}_{\rm int,g}/m_{\rm g}c^2 - \hat{H}_{\rm int,i}/m_{\rm i}c^2)$ can free-fall with a well-defined acceleration. Hence, even for an internal eigenstate, the CM will be in a superposition of states free-falling with a different acceleration (see Fig. 1a). As an example, take an eigenstate $|E_{\rm i}\rangle$ of $\hat{H}_{\rm int,i}$ localized around h: $|\Phi(0)\rangle = |E_{\rm i}\rangle|h\rangle$. To prepare $|E_{\rm i}\rangle$ one could use, for example, a mass spectroscope, selecting states with fixed inertial mass-energy. In free-fall, $|\Phi(0)\rangle$ evolves to $|\Phi(t)\rangle = \Sigma_k e^{-i\phi_k(t)}c_{ik}|\eta_k\rangle|h_k\rangle$, where $|\eta_k\rangle$ are eigenstates of $\hat{\eta}$, c_{ik} are normalized amplitudes defined by $|E_{\rm i}\rangle = \Sigma_k c_{ik}|\eta_k\rangle$, and $\phi_k(t)$ are free-evolution phases[36]. For $|h_k - h_j|$, $k \neq j$ greater than the coherence length, $|\Phi(t)\rangle$ becomes entangled (the CM position entangles to the internal DOF when position amplitudes become distinguishable, $\langle h_j|h_k\rangle \approx \delta_{jk}$). Testing for this entanglement would directly probe the quantum formulation of the WEP: separable states cannot evolve into entangled ones in any 'diagonal' test theory (with commuting internal energy operators).

Development of entanglement would here imply spatial modulations in the probability of finding the system at time t at height z, $P(z,t) = |\langle z|\Phi(t)\rangle|^2$ (Fig. 1b). In the regime of coherence length larger than $|h_j - h_k|$, the probability distribution would broaden along the gravity gradient. Importantly, these incoherent modulations or broadening cannot appear if $[\hat{H}_{\rm int,i}, \hat{H}_{\rm int,g}] = 0$ for an initial internal eigenstate unless the dynamics allows a pure state to evolve into a mixture $\hat{\rho}(t) := \Sigma_k |c_{ik}|^2 |\eta_k\rangle\langle\eta_k| \otimes |h_k\rangle\langle h_k|$. Such dynamics does not necessarily violate the WEP, but it explicitly violates unitarity of quantum mechanics. However, it still cannot account for the entanglement as $\hat{\rho}(t)$ is separable. Therefore, probing such modulations/broadening of the probability distributions of free-falling composite particles can test the quantum WEP under the assumption that unitarity of quantum theory is not violated.

Here we use data from an experiment with a Bose–Einstein condensate (BEC) of ^{87}Rb in free-fall[37] to bound the strength of quantum WEP violations. Non-vanishing variance of the parameter-matrix $\Delta\eta = \sqrt{\langle\eta^2\rangle - \langle\eta\rangle^2}$, taken in the state $|E_{\rm i}\rangle$, would mean anomalous broadening of the BEC cloud by $\Delta S \approx \Delta\eta g T^2/2$, where $T \approx 0.5$ s is the free-fall time and $g \approx 10\,{\rm m^{-2}}$. In the experiment, no modulations or spreading has been observed, so ΔS can be bounded by the size of the BEC $L \approx 10^{-4}$ m and $\Delta\hat{\eta} < 8 \times 10^{-5}$. Note that assuming unitarity of quantum mechanics and that atoms were initially in an eigenstate of $\hat{H}_{\rm int,i}$, having $\Delta\hat{\eta} \neq 0$ necessarily requires $[\hat{H}_{\rm int,i}, \hat{H}_{\rm int,g}] \neq 0$.

A quantum WEP test realized recently in the group of Tino[4] used a different approach, looking for excess phase noise in an atom interferometer with atoms prepared in a superposition of hyperfine energy levels, as compared with the phase noise in the experiment with atoms prepared in internal eigenstates. (It provided a bound on quantum WEP violations of order 10^{-8}.) At present, it is not feasible to perform interference experiments with atoms in superpositions of energy levels with a larger energy gap. An entanglement-based test could be feasible with a sufficiently long-lived internal superposition and could allow testing quantum WEP over larger energy scales than hyperfine splitting.

Testing the quantum formulation of LPI and LLI. Internal energy oscillations
Non-commuting internal energy operators would generally induce internal state oscillations, conceptually similar to the flavour oscillations in neutrinos. As a first example, consider that the WEP holds and thus $\hat{M}_{\rm i} = \hat{M}_{\rm g} \equiv \hat{M}$, but that LLI and LPI are violated, so $\hat{M} \neq \hat{M}_{\rm r}$. In particular, if $[\hat{H}_{\rm int,r}, \hat{H}_{\rm int,i}] \neq 0$, an initial eigenstate of $\hat{H}_{\rm int,r}$ (eigenstate of the internal energy when the CM is at rest in a gravity-free region) will in general evolve to a superposition of different eigenstates of $\hat{H}_{\rm int,r}$. The kinetic and potential part of the particle's action take the usual form[36] $S = \hat{M}\int dt\frac{1}{2}(\dot{x}(t)^2 - \phi(x(t))) \equiv \hat{M}\mathcal{K}t$ where $\mathcal{K}t = -gz_0 t + \frac{1}{3}g^2t^3$ for a particle free-falling in a potential gz for time t, and initially localized around z_0. Without loss of generality, we can write $\hat{H}_{\rm int,r} = mc^2\begin{pmatrix} 0 & 0 \\ 0 & e_{\rm r}\end{pmatrix}$ and parameterize $\hat{M} = m\begin{pmatrix} 1 & \alpha_{\rm q} \\ \alpha_{\rm q}^* & 1+e\end{pmatrix}$. Here $\alpha_{\rm q} = 0$ and $e_{\rm r} \neq$ e would give a classical-EEP violation (rest internal energy eigenvalue $me_{\rm r}$ would be different from the contribution of internal energy to inertia and weight me. $\alpha_{\rm q} \neq 0$ entails non-commutativity of $\hat{H}_{\rm int,r}$ and $\hat{H}_{\rm int,i(g)}$, and thus a violation of the quantum-EEP. For an initial ground state $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ of $\hat{H}_{\rm int,r}$, the transition probability to the excited state $|E_{\rm r}\rangle = \begin{pmatrix} 0 \\ 1\end{pmatrix}$ is $P_{\rm oscill} = |\langle E_{\rm r}|0\rangle|^2$ and is given by

$$P_{\rm oscill} = \frac{2|\alpha_{\rm q}^2|\mathcal{K}^2}{\left(4|\alpha_{\rm q}^2|\mathcal{K}^2 + e_{\rm r}^2c^4\left(1 - \frac{e}{e_{\rm r}}\frac{\mathcal{K}}{c^2}\right)^2\right)^2}\left(1 - \cos\left(\frac{mt}{\hbar}\sqrt{4|\alpha_{\rm q}^2|\mathcal{K}^2 + e_{\rm r}^2c^4\left(1 - \frac{e}{e_{\rm r}}\frac{\mathcal{K}}{c^2}\right)^2}\right)\right) \tag{9}$$

(see Fig. 2).

Crucially, no oscillation occurs under classical-EEP violations: that is, when $\alpha_{\rm q} = 0$, even if $\alpha_{\rm c} := 1 - e_{\rm r}/e \neq 0$. Therefore, an experiment looking for such an effect could test the quantum formulation of the EEP. We note that an experiment measuring the average free-fall time (acceleration), with single atoms, is in principle only testing the WEP—any anomalous magnitude of the measured time (or acceleration) can be explained by $\hat{M}_{\rm g} \neq \hat{M}_{\rm i}$. However, an experiment comparing these free-fall times for atoms prepared in various internal states—including coherent superpositions of internal states with different relative phases—could test a quantum formulation of the WEP.

Under the assumption that the WEP holds, the BEC free-fall experiment[37] can bound $\alpha_{\rm q}$: atoms that would transition to a different internal state would add to incoherent background of the interference pattern (they will not be addressed by the lasers inducing Bragg transitions). The reported 40% interference contrast is, however, compatible with a maximal violation of the order of the rest mass of the atom $m\alpha_{\rm q} < 100$ GeV. (We note that SME coefficients for LLI violations are constrained in the range 10^{-20}–10^{-35} GeV.)

Internal and external state correlations The validity of LLI and LPI can be tested by probing special and general relativistic time dilation. The key observation here is that different internal Hamiltonians $\hat{H}_{\rm int,\alpha}$ generally lead to different rates of internal dynamics. If an internal DOF $\hat{q}$ evolves under $\hat{H}^{\rm Q}_{\rm test}$

$$\dot{\hat{q}}(\hat{Q}, \hat{P}) = \dot{\hat{q}}_{\rm r}\hat{I}_{\rm ext} - \dot{\hat{q}}_{\rm i}\frac{\hat{P}^2}{2m_{\rm i}^2c^2} + \dot{\hat{q}}_{\rm g}\frac{\phi(\hat{Q})}{c^2} \tag{10}$$

where $\hat{I}_{\rm ext}$ denotes identity on the space of external DOFs. Equation (10) can be conveniently written in terms of $\dot{Q}$, the canonically conjugate velocity to P (compare with Supplementary Note 3): $\dot{\hat{q}}(\hat{Q}, \dot{\hat{Q}}) = \dot{\hat{q}}_{\rm r}\hat{I}_{\rm ext} - \dot{\hat{q}}_{\rm i}\frac{\dot{\hat{Q}}}{2c^2} + \dot{\hat{q}}_{\rm g}\frac{\phi(\hat{Q})}{c^2}$. If the internal energy operators are equal, we have $\dot{\hat{q}}_\alpha = -i/\hbar[\hat{q}, \hat{H}_{\rm int}] =: \dot{\hat{q}}$ and $\dot{\hat{q}}(\hat{Q}, \hat{P}) = \dot{\hat{q}}\left(\hat{I}_{\rm ext} - \frac{\hat{P}^2}{2m_{\rm i}^2c^2} + \frac{\phi(\hat{Q})}{c^2}\right)$. This expresses the universality of special relativistic and gravitational time dilation, up to $\mathcal{O}(c^{-2})$.

In any theory in which internal energies are essentially classical, such as $H^{\rm C}_{\rm test}$ (6), EEP violations only cause shifts of the internal energy eigenvalues. Special relativistic shifts are universal if $E_{\rm r} = E_{\rm i}$, and the gravitational shifts are universal if $E_{\rm r} = E_{\rm g}$. The same conditions express LLI and LPI in a fully classical theory. In turn, any experiment measuring internal energy spectra or frequency shifts can only test the classical formulation of the EEP: any LLI or LPI violation thus revealed could be explained by commuting internal energies. Violations of the quantum formulation of LLI (LPI), occurring when $[\hat{H}_{\rm int,r}, \hat{H}_{\rm int,i}] \neq 0$ ($[\hat{H}_{\rm int,r}, \hat{H}_{\rm int,g}] \neq 0$), result in further effects originating from the fact that an eigenstate of, say, $\hat{H}_{\rm int,r}$ will generally not be an an eigenstate of $\hat{H}_{\rm int,i}$ ($\hat{H}_{\rm int,g}$). As an example, consider a Mach–Zehnder interferometer in which a particle travels along two semi-classical paths γ_1

and γ_2 in superposition (Fig. 3a), and their interference is measured. When the CM follows the path γ_j, $j=1, 2$, an effective Hamiltonian $\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_j)$ describes the dynamics of the internal DOFs along that path. Any initial eigenstate of $\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_1)$, denoted $|E(\gamma_1)\rangle$, is stationary along γ_1, evolving only by a phase, like 'rock'. For $[\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_1), \hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_2)] \neq 0$ the same state generally will evolve non-trivially along γ_2 like a 'clock'. The internal DOF becomes entangled with the CM, and coherence of the path superposition decreases.

For a quantitative estimation, let us focus first on LPI and consider paths γ_j so that the particle remains at a fixed height h_j for time T. Approximating gravity to homogeneous potential for simplicity, we obtain $\mathcal{V}=|\langle e^{\frac{i}{\hbar}\int_{\gamma_1} ds\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_1(s))} e^{-\frac{i}{\hbar}\int_{\gamma_2} ds\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_2(s))}\rangle|$. Vertical parts of γ_j can be arranged to contribute equally to the final state, and thus the CM coherence is given by $\mathcal{V}=|\cos\left(\Delta H_{\mathrm{int,g}}\frac{ghT}{\hbar c^2}\right)|$, where $h\equiv h_2$, $h_1=0$ and $\Delta H_{\mathrm{int,g}} := \sqrt{\langle\hat{H}^2_{\mathrm{int,g}}\rangle - \langle\hat{H}_{\mathrm{int,g}}\rangle^2}$; the expectation values are taken with respect to the initial state $|E(\gamma_1)\rangle$. The above $\mathcal{V}$ is also the interferometric visibility in such an experiment. The probability of registering the particle in the detector $D_\pm$ correspondingly reads $P_\pm=\frac{1}{2}\left(1\pm\cos\left(\Delta H_{\mathrm{int,g}}\frac{ghT}{\hbar c^2}\right)\cos\left(\left\langle\hat{M}_{\mathrm{g}}\right\rangle\frac{ghT}{\hbar}\right)\right)$. The first cosine comes from the overlap between the states following different paths, the second comes from their relative phase. For arbitrary γ_j and a generic initial state, the above generalizes to

$$P_\pm=\frac{1}{2}\pm\frac{1}{2}\mathrm{Re}\{\langle e^{\frac{i}{\hbar}\int_{\gamma_1} ds\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_1(s))}\times e^{-\frac{i}{\hbar}\int_{\gamma_2} ds\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_2(s))}\rangle\}$$

and

$$\mathcal{V}=|\langle e^{\frac{i}{\hbar}\int_{\gamma_1} ds\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_1(s))} e^{-\frac{i}{\hbar}\int_{\gamma_2} ds\hat{H}^{\mathrm{Q}}_{\mathrm{test}}(\gamma_2(s))}\rangle|.$$

Importantly, if the EEP holds, the visibility in an interference experiment with composite particles can be affected only if the initial state is not an internal energy eigenstate and if there is sufficient time dilation between the paths[5,6]. In a standard theory and in an ideal case, an internal energy eigenstate results in an interference pattern with maximal visibility $\mathcal{V}=1$. Similarly, according to a 'diagonal' test theory, an initial eigenstate remains an eigenstate (along any path), with at most an anomalous internal energy value. This yields modifications of the phase shift, but still allows maximal visibility. Therefore, modulations of the interference visibility for a system initially in its internal energy eigenstate can directly probe the quantum formulation of the EEP.

In a given experiment, what aspects of the EEP are tested is determined by the parameters that are actually measured. If the system is prepared in an eigenstate of $\hat{H}_{\mathrm{int,r}}$, it is meaningful to express $\Delta H_{\mathrm{int,g}}\equiv E_{\mathrm{r}}\Delta\alpha$ where $\Delta\alpha$ is a variance of $\hat{\alpha}=\hat{I}_{\mathrm{int}}-\hat{H}_{\mathrm{int,g}}\hat{H}^{-1}_{\mathrm{int,r}}$. In atom-fountain interference experiments, the path separation is given by atom–laser interactions: $h=\hbar k t_{\mathrm{s}}/\langle\hat{M}_{\mathrm{i}}\rangle$, where k is a wave-vector of the laser and t_{s} is the time between the laser pulses. Because $\frac{\langle\hat{M}_{\mathrm{g}}\rangle}{\langle\hat{M}_{\mathrm{i}}\rangle}\neq 1$ is consistent with $[\hat{M}_{\mathrm{g}}, \hat{M}_{\mathrm{i}}]=0$, we parameterize $\frac{\langle\hat{M}_{\mathrm{g}}\rangle}{\langle\hat{M}_{\mathrm{i}}\rangle}=1-\eta_{\mathrm{class}}$. Finally, we obtain the detection probabilities

$$P_\pm=\frac{1}{2}\pm\frac{1}{2}\cos\left(\Delta\alpha\frac{E_{\mathrm{r}}}{\langle M_{\mathrm{i}}\rangle c^2}gkt_{\mathrm{s}}T\right)\cos\left((1-\eta_{\mathrm{class}})gkt_{\mathrm{s}}T\right)$$

as qualitatively sketched in Fig. 3b). The factor $(1-\eta_{\mathrm{class}})$ stems from the gravitational phase shift and its modifications due to violations of the classical formulation of the WEP. Furthermore, an anomalous phase shift can be fully explained by a theory that violates the WEP only in its non-relativistic limit (with $m_{\mathrm{i}}\neq m_{\mathrm{g}}$ and $\hat{H}_{\mathrm{int,i}}=\hat{H}_{\mathrm{int,r}}=\hat{H}_{\mathrm{int,g}}$). Interferometric visibility—the first cosine—can test the quantum formulation of LPI as it depends on the variance of $\hat{\alpha}$.

Currently, quantum violations of LPI are not constrained by any direct experiment. Assuming the validity of LLI, approximate constraints can be obtained from, for example, an experiment comparing gravitational phase-shifts for ^{85}Rb and ^{87}Rb, and for different hyperfine states of ^{85}Rb, reported in ref. [21]. For rubidium, we can take $\langle\hat{M}_{\mathrm{r}}\rangle=\langle\hat{M}_{\mathrm{i}}\rangle\approx 2.5\times 10^{-25}$ kg, $E_{\mathrm{r}}=\hbar\omega$ where $\omega\approx 10^{15}$ Hz, and $k\approx 7.5\times 10^6$ m^{-1}. The time T was varied between 40.207 ms and 40.209 ms. We therefore set $T=T_0\pm\delta T$ with $T_0=40$ ms and $\delta T=10^{-3}$ ms. The path separation was $h\approx\hbar k T_0/\langle M_{\mathrm{r}}\rangle$ and the visibility was $\mathcal{V}=0.09$, constant over the time interval between $T_0\pm\delta T$. Visibility modulations can thus be upper-bounded by 10^{-3}, which finally yields $\Delta\alpha<9\times 10^6$. The high value of this bound highlights the need for dedicated tests of the EEP in quantum theory.

To verify the validity of LLI in quantum theory, an analogous set-up to the one in Fig. 3 can be used. Here the two paths would remain at the same height (gravitational potential), but one, say γ_1', would stay at rest relative to the laboratory, whereas the other, γ_2', would be given some velocity, then redirected back and overlapped with the first. If $[\hat{H}_{\mathrm{int,r}}, \hat{H}_{\mathrm{int,i}}]\neq 0$, an internal state stationary along γ_1' will not be stationary along γ_2' in general. Violations of the quantum formulation of LLI would therefore give rise to modulations of the interference contrast, whereas violations of its classical formulation would result in an anomalous phase shift.

Finally, we note that even if one assumes that only violations of the classical EEP are possible (that internal energy operators must commute), a quantum test theory of the EEP (with quantized internal energies) is still needed, for example to describe effects arising from classical violations of the WEP in an experiment in which a test-body is initially in a superposition of different internal mass-energy eigenstates[7].

Discussion of controversies around the EEP in quantum theory. The status of the equivalence principle in quantum physics is actively researched both from the experimental[17–21] and from the theoretical perspective[38–48]. There is also a growing interest in performing high-precision quantum tests of the EEP in future space-based missions[17,19,20]. This is largely motivated by an expectation that the EEP is violated as a low-energy consequence of quantum gravity effects[49–51]. However, the applicability and even the validity of the EEP in standard quantum formalism are still debated. One can find arguments that there is a tension between the formulation of the EEP and the quantum formalism[41]; that the EEP does not hold in quantum theory[43,44,52]; and that there is no difference between testing the EEP in classical and quantum physics as both theories use the same action for a test particle[48]. First, the equivalence hypothesis can be incorporated into any physical theory by writing it in arbitrary coordinates and postulating equivalence between accelerated coordinates and effects of homogeneous gravitational fields. In this sense, the hypothesis is incorporated into quantum theory[53]. In this context, it is also often pointed out that a classical notion of a trajectory does not strictly apply in quantum mechanics and that mass does not cancel from the time evolution of quantum states. Although both these statements are correct, they neither invalidate the formulation of the EEP in quantum theory nor entail that it is violated—see further discussion in Supplementary Note 4.

Second, when internal DOFs are quantized, the actions of a classical and of a quantum test particle are quantitatively different—and there is a corresponding difference in the conditions that need to be tested to conclude that the EEP holds in the two theories, as listed in Table 1. The difference originates from the fact that, in the classical limit, internal energy operators necessarily commute, whereas this has to be verified in quantum mechanics.

Third, it is indeed true that the quantum tests of the EEP realized thus far were conceptually equivalent to the classical ones (apart from ref. [4] based on the ideas developed in the present work): they probed the equivalence of masses and spectra, at most in combination with the validity of the superposition principle for the CM[18–20,48]. Quantum metrology techniques have only been used to realize EEP tests with lighter systems, at shorter distance scales and with a better control over the spin or energy than those achievable in 'classical' tests[54]. Further experiments are required to test the distinctive quantum aspects of the EEP, such as those sketched in this work or discussed in the ensuing proposals[55,56].

Data availability. The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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