

Prospect of Studying Nonperturbative QED with Beam-Beam Collisions

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We demonstrate the experimental feasibility of probing the fully nonperturbative regime of quantum electrodynamics with a 100 GeV-class particle collider. By using tightly compressed and focused electron beams, beamstrahlung radiation losses can be mitigated, allowing the particles to experience extreme electromagnetic fields. Three-dimensional particle-in-cell simulations confirm the viability of this approach. The experimental forefront envisaged has the potential to establish a novel research field and to stimulate the development of a new theoretical methodology for this yet unexplored regime of strong-field quantum electrodynamics.

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The interaction of light and matter is governed by quantum electrodynamics (QED), which is the most successfully tested theory in physics. According to the present understanding of QED, the properties of matter change qualitatively in the presence of strong electromagnetic fields. The importance of strong-field quantum effects is determined by the Lorentz invariant parameter $\chi = E^*/E_{\text{cr}}$ [1–4] (also called beamstrahlung parameter in the context of particle colliders), which compares the electromagnetic field in the electron or positron rest frame E^* with the QED critical field $E_{\text{cr}} = m^2 c^3 / (e\hbar) \approx 1.32 \times 10^{18}$ V/m. Here, m and e are the electron or positron mass and charge, c is the speed of light, and \hbar is the reduced Planck constant, respectively. Whereas classical electrodynamics is valid if $\chi \ll 1$, quantum effects like the recoil of individually emitted photons (quantum radiation reaction) and the creation of matter from pure light become important in the regime $\chi \gtrsim 1$ [1–4]. Eventually, the interaction between light and matter becomes fully nonperturbative if $\chi \gg 1$.

The behavior of matter near QED critical field strengths (i.e., the regime $\chi \sim 1$) is important in astrophysics (e.g., gamma-ray bursts, pulsar magnetosphere, supernova explosions) [5–7], at the interaction point of future linear particle colliders [8–15], and in upcoming high energy density physics experiments, where laser-plasma interactions will probe quantum effects [16]. Experimental investigations of strong-field QED have just approached $\chi \lesssim 1$, e.g., by combining highly energetic particles with intense optical laser fields. This experimental scheme, first realized in the SLAC E-144 experiment [17,18], has been recently revisited [19,20]. Notable alternatives are x-ray free electron lasers [21], highly charged ions [22], heavy-ion collisions

[23], and strong crystalline fields [24]. The success of QED in the regime $\chi \lesssim 1$ is based on the smallness of the fine-structure constant $\alpha \approx 1/137$, which facilitates perturbative calculations.

Inside an extremely strong electromagnetic background field, however, the situation changes profoundly. According to the Ritus-Narozhny conjecture the actual expansion parameter of QED in the strong-field sector $\chi \gg 1$ is $\alpha\chi^{2/3}$. [25–27]. Correspondingly, QED becomes a strongly coupled theory if $\alpha\chi^{2/3} \gtrsim 1$ and the so-called dressed loop expansion breaks down. This implies that the emission of a virtual photon by an electron or positron or the temporary conversion of a photon into a virtual electron-positron pair is no longer an unlikely event. Therefore, the existing theoretical framework is not suitable for this regime.

The fully nonperturbative sector ($\alpha\chi^{2/3} \gtrsim 1$) is currently seen as beyond experimental reach. The fundamental challenge in probing such extreme fields is the fast radiative energy loss by electrons or positrons. Its mitigation requires the switching time of the background field to be smaller than the electron or positron radiative lifetime $\tau_l \sim \gamma\tau_c / (\alpha\chi^{2/3})$ ($\tau_c = \tilde{\lambda}_c / c \approx 1.3 \times 10^{-21}$ s; $\tilde{\lambda}_c = \hbar / (mc) \approx 3.9 \times 10^{-13}$ m and γ denotes the Lorentz gamma factor) [28]. As the spatial extent of an optical laser pulse must be at least of the order of the laser wavelength $\lambda_l \sim \mu\text{m}$, we need a multiple TeV electron or positron beam ($\gamma \sim 10^7$) to ensure $\lambda_l \lesssim \gamma\tilde{\lambda}_c$. Therefore, reaching the regime $\alpha\chi^{2/3} \gtrsim 1$ with electron-optical laser collisions is not viable at the 100 GeV scale.

In this Letter, we show that using tightly compressed and focused beams it is possible to probe for the first time the

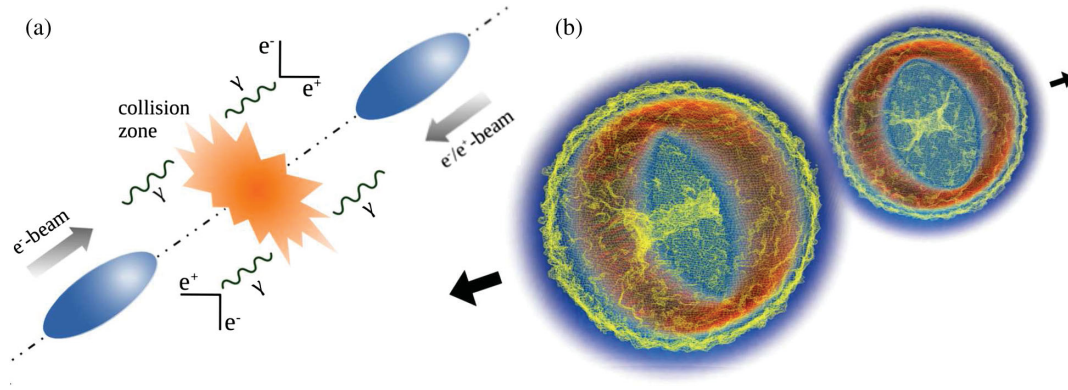


FIG. 1. (a) Illustration of a beam-beam collider for probing the fully nonperturbative QED regime. (b) 3D OSIRIS-QED simulation of the collision of two spherical 10 nm electron beams with 125 GeV energy (blue). The fully nonperturbative QED regime $\alpha\chi^{2/3} \geq 1$ is experienced by 38% of the colliding particles (red). The interaction produces two dense gamma-ray beams with 0.2 photons with $E_\gamma \geq 2mc^2$ per primary electron (yellow).

fully nonperturbative QED regime with a 100 GeV-class particle collider (Fig. 1). We argue that these beams could be produced with accessible technology. Full 3D particle-in-cell (PIC) simulations confirm the possibility of limiting beam energy losses to $\lesssim 5\%$, implying that the majority of particles reach the strong field region.

To estimate the importance of nonperturbative effects, we take phenomenologically into account that quantum fluctuations dynamically increase the effective electron or positron mass and thus the effective QED critical field. As a result, one expects that radiation and pair production are attenuated with respect to the perturbative predictions. Our simulations show that corrections on the order of 20%–30% are to be expected (see below). Correspondingly, nonperturbative effects should be observable with a 100 GeV-class particle collider.

The breakdown of perturbation theory in the regime $\alpha\chi^{2/3} \gtrsim 1$ has an intuitive explanation. In vacuum, the characteristic scales of QED are determined by the electron or positron mass m . In the presence of a background field, however, the fundamental properties of electrons, positrons, and photons are modified by quantum fluctuations (Fig. 2). Figuratively speaking, the quantum vacuum is not empty but filled with virtual electron-positron pairs. A strong electromagnetic field polarizes or ionizes the vacuum, which therefore behaves like an electron-positron pair plasma [29]. As a result, the “plasma frequency of the vacuum” changes the photon dispersion relation, implying that a photon acquires an effective mass $m_\gamma(\chi)$, see Supplemental Material [30]. The appearance of a photon mass induces qualitatively new phenomena like vacuum birefringence and dichroism [41–44]. Perturbation theory is expected to break down in the regime $m_\gamma(\chi) \gtrsim m$, where modifications due to quantum fluctuations become of the same order as the leading-order tree-level result (Fig. 2).

In order to provide an intuitive understanding for the scaling of $m_\gamma(\chi)$, a photon with energy $\hbar\omega_\gamma \gg mc^2$ is

considered, which propagates through a perpendicular electric field with magnitude E in the laboratory frame. The quantum parameter χ associated with this photon is $\chi \sim \gamma E/E_{\text{cr}}$, where $\gamma = \hbar\omega_\gamma/(mc^2)$ can be interpreted as a generalized Lorentz gamma factor. As the polarization of the quantum vacuum requires at least two interactions (Fig. 2), it is expected that $m_\gamma^2(\chi) \sim \alpha M^2$ (the plasma frequency of a medium exhibits the same scaling in α). Here, $M \sim eE\Delta t/c$ denotes the characteristic mass scale induced by the background field and Δt represents the characteristic lifetime of a virtual pair.

The scaling of Δt is determined by the Heisenberg uncertainty principle $\Delta t\Delta\epsilon \sim \hbar$, where $\Delta\epsilon = \epsilon_- + \epsilon_+ - \epsilon_\gamma$ quantifies energy nonconservation at the pair production vertex. Here, $\epsilon_- \approx \epsilon_+ = \sqrt{(pc)^2 + m^2c^4 + (eE\Delta tc)^2} \approx pc + (eE\Delta tc)^2/(2pc)$ are the electron or positron energies and $\epsilon_\gamma = p_\gamma c$ is the energy of the gamma photon (the electron and positron have the same initial momentum $p = p_\gamma/2$ at threshold). Assuming, $\chi \gg 1$ and thus $eE\Delta t \gg mc$

$$\begin{aligned} \frac{\mathcal{P}}{m^2} &= \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots \\ &\sim \alpha\chi^{2/3} \quad \sim \alpha^2\chi^{2/3}\log\chi \quad \sim \alpha^3\chi\log^2\chi \quad \sim \alpha^n\chi^{(2n-3)/3} \\ &\text{Narozhny 1968} \quad \text{Morozov 1977} \quad \text{Narozhny 1980} \quad \text{conjecture} \\ \frac{\mathcal{M}}{m} &= \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots \\ &\sim \alpha\chi^{2/3} \quad \sim \alpha^2\chi\log\chi \quad \sim \alpha^3\chi^{5/3} \quad \sim \alpha^n\chi^{(2n-1)/3} \\ &\text{Ritus 1970} \quad \text{Ritus 1972} \quad \text{Narozhny 1980} \quad \text{conjecture} \end{aligned}$$

FIG. 2. Dressed loop expansion of the polarization operator \mathcal{P} (top row) and mass operator \mathcal{M} (bottom row). Wiggly lines denote photons and double lines dressed electron or positron propagators [1]. According to the Ritus-Narozhny conjecture, the diagrams shown represent the dominant contribution at n loop and $\alpha\chi^{2/3}$ is the true expansion parameter of strong-field QED in the regime $\chi \gg 1$ [25–27].

(momentum acquired by the charges in the background field E), we find $\Delta\epsilon \sim (eE\Delta t/c)^2/(\hbar\omega_\gamma)^2$. Notably, the resulting field-induced mass scale $M \sim eE\Delta t/c \sim m\chi^{1/3}$ is independent of m (note that $\chi \sim m^{-3}$). This suggests a new regime of light-matter interaction, where the characteristic scales of the theory are determined by the background field ($M \gg m$). The scaling $m_\gamma^2(\chi) \sim \alpha M^2 \sim \alpha\chi^{2/3}m^2$ in the regime $\chi \gg 1$ implies $m_\gamma \gtrsim m$ if $\alpha\chi^{2/3} \gtrsim 1$ and thus a breakdown of perturbation theory at the conjectured scale [25–27]. The same scaling is also found for the electron or positron effective mass by analyzing the mass operator (see Supplemental Material [30]).

A similar breakdown of perturbation theory is predicted for supercritical magnetic fields [$B \gg B_{\text{cr}} = m^2c^3/(e\hbar) \approx 4.41 \times 10^9$ T]. Whereas the mass correction for electrons in the lowest Landau level scales logarithmically [45], photons acquire an effective mass via the polarization operator, which exhibits a power-law scaling [46,47] (for a discussion of possible astrophysical observables see, e.g., Ref. [48]). For supercritical magnetic fields, effective dimensional reduction facilitates nonperturbative calculations [49–52]. Note that the case considered here is complementary and qualitatively different, as it corresponds on the contrary to ultrarelativistic electrons or positrons occupying very high Landau levels. As a result, they can emit photons and produce pairs and thus provide two accessible observables which are affected by radiative corrections.

The key challenge for reaching the fully nonperturbative regime $\alpha\chi^{2/3} \gtrsim 1$ in beam-beam collisions is the mitigation of radiative losses through beamstrahlung: the emission of radiation as the colliding particles are bent in the fields of the opposing bunch. This process is characterized by four beam parameters: the transverse σ_r and the longitudinal σ_z dimensions of the bunches ($\sigma_r = \sigma_x = \sigma_y$ for radially symmetric beams), the number of particles per bunch N (i.e., the total charge), and the beam Lorentz factor γ . Lorentz invariance requires that only the ratio $\sigma_z^* = \sigma_z/\gamma$ is relevant, implying three independent degrees of freedom.

The total radiation probability W (per beam particle) and the disruption parameter D , which characterizes the transverse motion of the beam particles, scale as

$$W \sim \alpha\chi_{\text{av}}^{2/3} \frac{\sigma_z^*}{\lambda_c}, \quad D \sim \frac{N\alpha\lambda_c\sigma_z^*}{\sigma_r^2}, \quad \chi_{\text{av}} \approx \frac{5}{12} \frac{N\alpha\lambda_c^2}{\sigma_r\sigma_z^*}, \quad (1)$$

where χ_{av} denotes the average value of the beamstrahlung parameter χ (in the accelerator science literature the symbol $\Upsilon = \chi_{\text{av}}$ is commonly used). The given estimate for χ_{av} holds for a radially symmetric Gaussian charge density profile [9]. In order to achieve a controlled interaction $D \ll 1$ is desirable, which implies that the classical trajectories of the colliding particles are only slightly distorted.

TABLE I. Comparison between the parameters of the non-perturbative QED (NPQED) collider discussed here and other existing linear accelerator or collider designs. Collision parameters for FACET-II [53] are not applicable, as it has only one beam. Here χ_{av} and χ_{max} for ILC [54] and CLIC [55] are calculated without taking into account the expected change in the beam size during collision, which is characteristic for high disruption parameters.

Parameter	[Unit]	NPQED			
		Collider	FACET-II	ILC	CLIC
Beam energy	[GeV]	125	10	250	1500
Bunch charge	[nC]	0.14–1.4	1.2	3.2	0.6
Peak current	[kA]	1700	300	1.3	12.1
Energy spread (rms)	[%]	0.1	0.85	0.12	0.34
Bunch length (rms)	[μm]	0.01–0.1	0.48	300	44
Bunch size (rms)	[μm]	0.01	3	0.47	0.045
Pulse rate \times	[Hz] \times	100 \times	30 \times	5 \times	50 \times
Bunches/pulse	N_{bunch}	1	1	1312	312
Beamstrahlung Parameter	χ_{av}	969		0.06	5
Disruption Parameter	χ_{max}	1721		0.15	12
Disruption Parameters	$D_{x,y}$	0.001–0.1		0.3	0.15
Peak electric field	[TV/m]	4500	3.2	0.2	2.7
Beam power	[MW]	0.002–0.02	10^{-4}	5	14
Luminosity	[$\text{cm}^{-2} \text{s}^{-1}$]	6×10^{30} -4×10^{32}		10^{34}	10^{34}

The requirements given above ($\alpha\chi^{2/3} \gtrsim 1$, $D \lesssim 0.01$, and $W \lesssim 1$) constrain the three beam parameters: $N \gtrsim 1/\alpha^4 \sim 10^9$ (i.e., $\gtrsim 0.1$ nC per bunch), $\sigma_r \sim 10\sqrt{N}\alpha\lambda_c \sim 10$ nm, and $\sigma_z^* \lesssim \lambda_c$. For a beam energy of ≈ 100 GeV ($\gamma \approx 2 \times 10^5$) this implies $\sigma_z \lesssim 100$ nm. In general, decreasing σ_z is beneficial for all three parameters (χ , D , W), whereas increasing the charge must be accompanied by a transverse compression to keep the disruption parameter small. According to these considerations, the natural set of parameters for a ~ 100 GeV nonperturbative QED (NPQED) collider, which is capable of reaching $\alpha\chi^{2/3} \gtrsim 1$ with low disruption, is given in Table I.

The NPQED collider discussed here maximizes the beam fields by employing highly compressed and round bunches. This approach differs significantly from existing linear collider designs like ILC [54] or CLIC [55], which use flat ($\sigma_x/\sigma_y > 10$) beam configurations to minimize self-generated fields for a given luminosity requirement. The idea for this NPQED collider originated from SLAC's FACET-II [53], designed to generate beams with up to 300 kA peak current ($\sigma_z \sim 0.5 \mu\text{m}$) at 10 GeV energy. Merging the high energy, high transverse quality beams of linear collider designs with the high peak compression of FACET-II encapsulates the key design challenges (Table I).

Nonperturbative QED can be probed with either electron-electron or electron-positron collisions. Using only electrons is preferable, as it avoids the challenge of generating positrons with the required longitudinal brightness. Next-generation cryogenic photoinjectors [56] aim for a factor > 4 improvement in emittance ($\sim 35 \text{ nm} \cdot \text{rad}$ at 100 pC). This will translate into electron focusing requirements similar to CLIC.

In order to obtain a compact accelerator design, high gradient technology (e.g., X -band radio frequency or plasma-based acceleration) could be employed, leading to an accelerating section with a footprint comparable with the SLAC linac. The fully nonperturbative QED regime can be studied with a single bunch per pulse at $\sim 100 \text{ Hz}$. An ILC-type linac and repetition rate might be required for compression stability and feedback systems. This would result in a luminosity equivalent to ILC at a much lower beam power.

The required bunch compression extends the state-of-the-art FACET-II design by a factor of 5. The anticipated increase of collective effects during bunch compression can be compensated by using advanced mitigation strategies, e.g., based on coherent synchrotron radiation (CSR) suppression and/or shielding techniques [57,58].

The final focus system can be based on the CLIC design, as the requirements are similar. However, delivering round beams with the required chromaticity compensation presents a unique challenge, especially when coherent effects from short bunches are considered. Alternatively, plasma focusing technology can be explored, as proposed in Ref. [59] and subsequently tested in multiple experimental facilities [60–62].

Even though a complete engineering design of the accelerator layout requires further R&D on the various subsystems—including high brightness beam sources, advanced beam compression techniques, final focus and beam delivery system—the NPQED collider parameters (Table I) rely only on evolutionary improvements of existing technology.

In order to confirm the possibility of reaching the regime $\alpha\chi^{2/3} \gtrsim 1$ with short, high current, colliding beams, we have performed 2D and 3D PIC simulations for the parameters of the NPQED collider in Table I. We employed the massively parallel, fully relativistic and electromagnetic PIC code OSIRIS-QED [63–66], which accounts self-consistently for the classical and the QED interaction between particles and fields (see Supplemental Material [30]). The latter is taken into account by employing photon emission and pair production probabilities inside a constant field [67–75]. This so-called local constant field approximation (LCFA) is applicable here, as the formation length $l_f = c\Delta t \sim \gamma\lambda_c/\chi^{2/3} \sim 1 \text{ nm}$ [10,76,77] is much smaller than the scale on which the field changes ($\sigma_z = 10 \text{ nm}$).

Figures 1 and 3 illustrate the results of 3D simulations for electron beams with $\sigma_z = 10 \text{ nm}$ ($D = 10^{-3}$). The

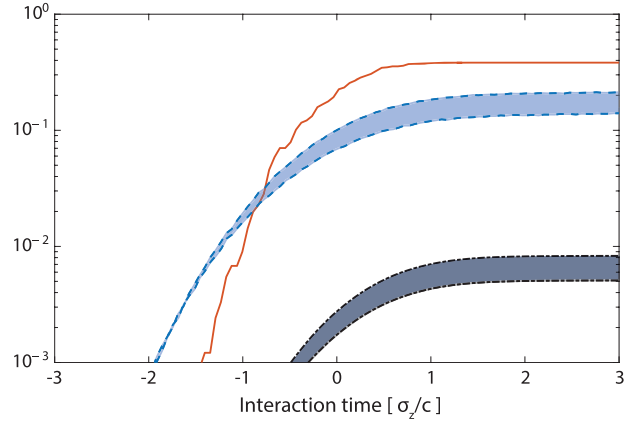


FIG. 3. Results of 3D simulations with OSIRIS-QED for the parameters of the NPQED collider in Table I. f_q (solid red line): fraction of beam electrons experiencing $\alpha\chi^{2/3} \geq 1$; f_γ (dashed blue line): number of high-energy photons ($E_\gamma \geq 2mc^2$) emitted per beam electron; f_p (dotted black line): number of secondary pairs per beam electron (all quantities are shown as a function of the beam crossing time). The shaded areas represent an estimate for the importance of nonperturbative quantum effects ($f_\gamma \sim 30\%$, $f_p \sim 25\%$). They were obtained by modifying the photon emission or pair production probabilities in OSIRIS-QED as explained in the main text and the Supplemental Material [30] (upper curves: state-of-the-art simulation, lower curves: modified probabilities).

simulations confirm that these beam parameters provide a suitable configuration to probe the fully nonperturbative QED regime, as a large fraction of beam electrons (38%) experience $\alpha\chi^{2/3} \geq 1$, while the beam energy losses are limited to $\lesssim 5\%$ (Fig. 4).

To estimate the importance of nonperturbative effects, we have phenomenologically taken into account that quantum corrections dynamically increase the effective electron or positron mass m_* (see Supplemental Material [30])

$$m_*^2 = m^2 + \delta m^2, \quad \delta m^2 \approx 0.84\alpha\chi^{2/3}m^2. \quad (2)$$

Thus, we phenomenologically replace χ by $\chi_* = \chi(m \rightarrow m_*)$ in the photon emission probability $W(\chi)$, i.e., employ $\tilde{W}(\chi) = W[\chi_*(\chi)]$ instead. After an elementary calculation one obtains

$$\chi_*^{2/3}(\chi) = \chi^{2/3}[1 + 0.84\alpha\chi^{2/3}]^{-1}. \quad (3)$$

Note that corrections to the effective electron or positron mass are not the only possible consequence of the nonperturbative regime. However, a complete and rigorous nonperturbative calculation, e.g., by employing methods developed for strongly coupled quantum field theories like QCD, is far beyond the scope of this Letter (e.g., truncated Schwinger-Dyson equations, resummation of certain diagram classes, renormalization-group techniques [78–80]).

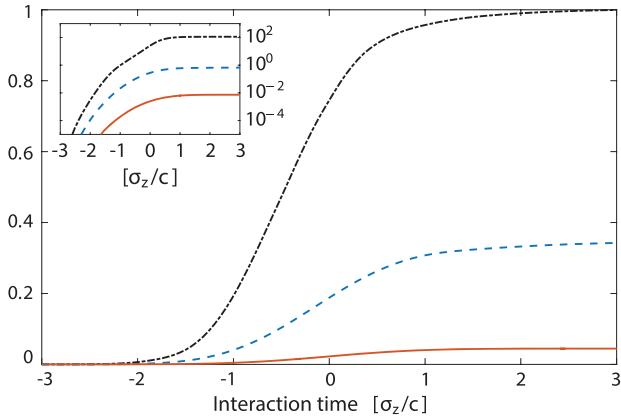


FIG. 4. Relative energy loss and number of secondary pairs f_p (inset plot) for three beam lengths: 10 nm (solid red line, $D = 10^{-3}$), 100 nm (dashed blue line, $D = 0.1$), and 500 nm (dotted black line, $D = 2.5$). In the latter case electron trajectories are severely modified, leading to strong disruption. The interaction results in a full-featured QED cascade, which multiplies the particle density and completely absorbs the energy of the initial beam.

However, the above estimate allows us to anticipate the order of magnitude of nonperturbative corrections in the regime $\alpha\chi^{2/3} \lesssim 1$ (Fig. 3), similar as a phenomenological recoil correction to the classical probabilities allows us to estimate the order of magnitude of quantum corrections in the regime $\chi \lesssim 1$ (see Supplemental Material [30]).

The bunch length, and correspondingly the disruption parameter, significantly impact the dynamics of the beam-beam interaction. To quantify this, a series of 2D simulations for $\sigma_z = 10\text{--}500$ nm ($D = 10^{-3} - 2.5$, Fig. 4) has been performed. The results indicate that electron trajectories become considerably disrupted for $D \gtrsim 0.1$ and energy losses are no longer negligible ($>30\%$). Therefore, $D < 0.1$ is preferable, as it provides a clean experimental interaction for testing theoretical nonperturbative QED predictions. However, we note that the $D \gtrsim 1$ regime represents an interesting scientific frontier, where the interplay between collective and strong-field quantum processes determines the evolution of the system.

In summary, we have shown that the collision of tightly compressed and focused 100 GeV-class electron beams would offer a very promising configuration for probing the fully nonperturbative QED regime $\alpha\chi^{2/3} \gtrsim 1$. The physics above this threshold remains completely unexplored experimentally, and there is no theoretical framework to describe light-matter interaction at such extreme fields. Investigations of this qualitatively different regime, both theoretical and experimental, are bound to discover new physical phenomena and advance the understanding of nonperturbative physics at the field intensity frontier.

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