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# A model of quantum collapse induced by gravity

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**Abstract.** We discuss a model where a spontaneous quantum collapse is induced by the gravitational interactions, treated classically. Its dynamics couples the standard wave function of a system with the Bohmian positions of its particles, which are considered as the only source of the gravitational attraction. The collapse is obtained by adding a small imaginary component to the gravitational coupling. It predicts extremely small perturbations of microscopic systems, but very fast collapse of QSMDS (quantum superpositions of macroscopically distinct quantum states) of a solid object, varying as the fifth power of its size. The model does not require adding any dimensional constant to those of standard physics.

A well-known difficulty in quantum mechanics is that the dynamical equations (Schrödinger or von Neumann equations) seem to predict the possible occurrence of quantum superpositions of macroscopically distinct states (QSMDS) [1,2] that are never observed, for instance the creation of Schrödinger cats [3,4]. To solve the difficulty, von Neumann [5,6] suggested to introduce a quantum collapse postulate, which is nowadays part of most introductory textbooks on quantum mechanics.

Several authors have proposed to relate quantum collapse to the effects of gravity. One can for instance assume that gravity is the source of a random noise acting on the state vector, and that this noise projects QSMDS onto one of its components localized in space. In references [7,8], Diosi discusses the introduction of a stochastic term in the Schrödinger (or von Neumann) equation that efficiently destroys QSMDS; see also references [9,10]. In reference [11], Penrose notes that the energy difference associated with different mass distributions leads to a violation of energy conservation, and suggest that this violation is spontaneously canceled by some random projection mechanism. Other contributions may be found in references [12–15]. Reviews of this class of theories can be found in Section 3-B of reference [16] and in reference [17].

Here we propose a model of the quantum dynamics that also provides a collapse, but with equations that are completely deterministic; gravity is treated as a classical field originating from the Bohmian positions of the particles. In classical physics, gravity already plays a special role, since it determines the curvature of space-time. In our model, we attribute to gravity another special feature, which is to introduce small non-Hermitian component in the evolution equation of the state vector. Nothing in this model is stochastic; the only source of randomness is the initial randomness of the Bohmian position, as in the de Broglie-Bohm (dBB) theory [18–23]. This model is in the line of a general view where space-time remains classical, and where the source of the curvature of space-time is the Bohmian positions of the particles; the various quantum fields propagate inside this classical space-time frame.

Combining elements from dBB and spontaneous collapse [24,25] theories is not a completely new idea. Reference [26] proposes to localize the wave function around the Bohmian positions, but with no real change of the Schrödinger dynamics; moreover, gravity plays no role in the localization process. References [27,28] consider a back action of the Bohmian positions on the wave function, but with a stochastic term, as in standard spontanous collapse theories [24,25].

For the sake of simplicity, here we discuss only spinless non-relativistic particles (including spins within a Pauli theory is nevertheless not particularly difficult). As in references [29,30], we use a dynamics involving an "expanded description" of the physical system: to the standard wave function $\Psi$ defined in the configuration space, we add (in the same space) a mathematical point $Q$, whose coordinates are determined by the Bohmian positions $q_n$ of all its particles. Incidentally, and in contrast with the usual interpretations of the de Broglie-Bohm (dBB) theory, we make no particular assumption concerning the physical reality of these positions; they can be seen, either as physically real, or as a pure mathematical object appearing in the dynamical equations.

The equations of this dynamics are given in Section 1. In Section 2 we discuss the predictions of the model in various situations, showing that it introduces no significant change for microscopic systems while it rapidly projects QSMDS onto one of its localized components. A conclusion is given in Section 3.

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## 1 Modified quantum dynamics

We assume that the Hamiltonian $H$ of a physical system is the sum of its internal Hamiltonian $H_{\text{int}}$ (including the kinetic energy of the particles and their mutual interactions) and of a gravitational Hamiltonian $H_G$, due to the attraction of external masses with mass density $n_G(\mathbf{r})$:

$$H = H_{\text{int}} + H_G \tag{1}$$

with:

$$H_G = -gGm \int \mathrm{d}^3r\ \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) \int \mathrm{d}^3r'\ \frac{1}{|\mathbf{r}-\mathbf{r}'|} n_G(\mathbf{r}') \tag{2}$$

In this relation, $g = 1$ in standard theory, $G$ is Newton's constant, $m$ the mass of the particles, and $\Psi(\mathbf{r})$ the quantum field operator of the particles contained in the physical system. With external sources of gravity, this Hamiltonian is completely standard. If $n_G(\mathbf{r}')$ is set equal to quantum local density average $\langle \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\rangle$, we obtain the usual Schrödinger-Newton equation [31].

### 1.1 Evolution of the state vector

We now leave standard quantum mechanics by making two non-standard assumptions. First, we assume that $H_G$ actually describes the internal gravitational attraction of the system, and that $n_G(\mathbf{r})$ is determined by the Bohmian positions $q_n$ of the $N$ particles of the system:

$$n_G(\mathbf{r}) = m \sum_{n=1}^{N} \delta(\mathbf{r} - \mathbf{q}_n) \tag{3}$$

Incidentally, one could also perform a spatial average over a distance $a_L$, as usual in GRW and CSL [24,25] theories, and write for instance:

$$n_G(\mathbf{r}) = \frac{m}{\pi^{3/2} a_L^3} \sum_{n=1}^{N} \mathrm{e}^{-(\mathbf{r}-\mathbf{q}_n)^2/\alpha_L^2} \tag{4}$$

Nevertheless, in what follows, we will only use the simpler form (3). Similarly it has been proposed in reference [32] to study chemical reactions (within standard quantum mechanics) by an approximation where the nuclei are treated classically, and where the backreaction of the quantum electrons on the nuclei is obtained by sampling the Bohmian positions of the electrons over their quantum distribution.

Second, as in reference [33], we assume that the dimensionless constant $g$ has a small imaginary part $\varepsilon$:

$$g = 1 - i\varepsilon \tag{5}$$

(one could choose any small number, for instance $\varepsilon = \alpha$, the fine structure constant). This introduces an antiHermitian part in $H_G$:

$$H_G = H_G^0 + iL \tag{6}$$

where $H_G^0$ is the Hermitian part of $H_G$:

$$H_G^0 = H_G(\varepsilon = 0) \tag{7}$$

and where $L$ is the localization operator:

$$L = \varepsilon Gm \int \mathrm{d}^3r\ \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) \int \mathrm{d}^3r'\ \frac{1}{|\mathbf{r}-\mathbf{r}'|} n_G(\mathbf{r}') \tag{8}$$

This operator is diagonal in the position representation. It is the second quantized form of the sum of $N$ single particle potentials taking large values in the vicinity of the Bohmian positions, in the regions of space where the gravitational attraction by these positions is strong. With (3), this expression becomes:

$$L = \varepsilon Gm^2 \int \mathrm{d}^3r\ \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) \sum_{n=1}^{N} \frac{1}{|\mathbf{r}-\mathbf{q}_n|} \tag{9}$$

In [29] we introduced a localization operator where the quantum operator $\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$ is coupled to the Bohmian positions with a Gaussian spatial average of range $a_L$. Here the Gaussian spreading function is replaced by a gravitational type of coupling that is proportional to the inverse distance.

The state vector $|\Phi(t)\rangle$ evolves according to:

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\Phi(t)\rangle = [H_{\text{int}} + H_G] |\Phi(t)\rangle \tag{10}$$

If $\varepsilon \neq 0$, the norm of $|\Phi(t)\rangle$ does not remain constant. We can nevertheless introduce the normalized ket $|\overline{\Phi}(t)\rangle$:

$$|\overline{\Phi}(t)\rangle = \frac{1}{\sqrt{\langle \Phi(t)\ |\Phi(t)\rangle}} |\Phi(t)\rangle \tag{11}$$

and set:

$$D_\Phi(\mathbf{r}) = \langle \overline{\Phi}(t)|\ \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\ |\overline{\Phi}(t)\rangle \tag{12}$$

This normalized state then evolves according to:

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\overline{\Phi}(t)\rangle = \Big[H_{\text{int}} + H_G^0 + i\varepsilon Gm \int \mathrm{d}^3r \int \mathrm{d}^3r' \\ \left[\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) - D_\Phi(\mathbf{r})\right]\ \frac{1}{|\mathbf{r}-\mathbf{r}'|}\ n_G(\mathbf{r}')\Big] |\overline{\Phi}(t)\rangle \tag{13}$$

To summarize, the two non-standard ingredients of our model are:

- the use of the Bohmian positions to define a density of matter in ordinary space; this density is the source of the classical gravitational field involving the usual Newton constant $G$.
- the introduction of a small imaginary part in $G$, so that the dynamics becomes irreversible and collapses QSMDS, as we see below.

### 1.2 Evolution of the Bohmian positions

We assume that the Bohmian positions $\mathbf{q}_n$ evolve according to the usual Bohmian equation of motion:

$$\frac{d\mathbf{q}_n(t)}{dt} = \frac{\hbar}{m} \overrightarrow{\nabla}_n \xi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \tag{14}$$

where $\xi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ is the phase of the wave function $\Phi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$, and $\overrightarrow{\nabla}_n$ the gradient taken with respect to $\mathbf{r}_n = \mathbf{q}_n$. Equivalently, this equation can also be written:

$$\frac{d\mathbf{q}_n(t)}{dt} = \frac{\hbar}{im\, D_\Phi(\mathbf{r})} \left\langle \overline{\Phi}(t) \right| \Psi^\dagger(\mathbf{r})\ \boldsymbol{\nabla}_\mathbf{r}\Psi(\mathbf{r}) - \boldsymbol{\nabla}_\mathbf{r}\Psi^\dagger(\mathbf{r})\ \Psi(\mathbf{r}) \left| \overline{\Phi}(t) \right\rangle \tag{15}$$

The condition of "quantum equilibrium" means that, when averaged over many realizations of an experiment, the distribution of the Bohmian positions in configuration space coincides with the modulus square of the wave function. In standard dBB theory with the usual Schrödinger equation, if this condition is satisfied at the initial time, it is also satisfied at any time. But this property no longer holds in our case, since we have modified the dynamics of the wave function. Nevertheless, in reference [30] we discuss why the relaxation process studied by Towler, Russell and Valentini [34,35] should ensure that this condition is still valid to an excellent approximation, except in a very short transient time during the appearance (and almost immediate collapse) of a QSMDS.

## 2 Discussion

We now discuss the effect of the localization term on the state vector. The situation is similar to that already considered in references [29,30] except that, here, the time constants of the collapse mechanism arise from a gravitational energy coupling the quantum particles with their Bohmian positions. We discuss only the simpler version (3) of the model, which introduces no fundamental parameter $a_L$, but similar conclusions apply as well if a non-zero value of $a_L$ is chosen.

### 2.1 Negligible effects on microscopic systems

Consider first the non-relativistic Schrödinger equation of the electron and proton in a Hydrogen atom, ignoring the spins for the sake of simplicity. Each of the two particles is subjected to two attractions:

– the usual Coulomb attraction, which introduces the usual two-body potential in the Schrödinger equation for the wave function.
– the gravitational attraction, appearing as a one-body attractive potential towards the position of an additional variable: the electron is attracted towards the Bohmian position $\mathbf{q}_p$ of the proton, and conversely the proton is attracted towards the Bohmian position $\mathbf{q}_e$ of the electron.

The ratio $X$ between the Coulomb and gravitational interactions is very large:

$$X \simeq \frac{q^2}{4\pi\varepsilon_0} \frac{1}{Gm_e m_p} \simeq 10^{39} \tag{16}$$

where $\varepsilon_0$ is the permittivity of vacuum, $q$ the electronic charge, $m_e$ the mass of the electron and $m_p$ the mass of the proton. This enormous value of $X$ ensures that the gravitational component plays no role in practice: we just recover the well-known fact that the gravitational attraction remains completely negligible in the Hydrogen atom. The divergences of $n_G(\mathbf{r})$ when $\mathbf{r} = \mathbf{q}_n$ do not create any special problem: as in the standard theory of the Hydrogen atom, they only introduce kinks in the wave function, but these kinks are $10^{29}$ times less pronounced than those introduced by the Coulomb potential; in practice, they have no effect. Moreover, the statistical distribution of $\mathbf{q}_p$ and $\mathbf{q}_e$ over many realizations coincides with the corresponding quantum distributions. Clearly, changing in this way the center of gravitational attraction has no practical consequence. In addition to this change, the model introduces a small imaginary component to the gravitational part of the Hamiltonian, which introduces an even more negligible perturbation.

Another example illustrates why, in most cases, the localization term has a very small effect. If $\left|\overline{\Phi}(t)\right\rangle$ is an eigenstate of the Hamiltonian $H_{\text{int}} + H_G^0$, the average energy $\left\langle H_{\text{int}} + H_G^0 \right\rangle$ remains constant:

$$\frac{\mathrm{d}}{\mathrm{d}t} \left\langle H_{\text{int}} + H_G^0 \right\rangle = 0 \tag{17}$$

More generally, if $\left|\overline{\Phi}(t)\right\rangle$ is an eigenstate of $A$ at time $t$, the localization term has no effect on the derivative of the average value of $A$ at time $t$:

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\text{loc}} \left\langle \overline{\Phi}(t) \right| A \left| \overline{\Phi}(t) \right\rangle = 0 \tag{18}$$

This is because, if $a$ is the eigenvalue of $A$, we have:

$$\begin{aligned}\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\text{loc}} \left\langle \overline{\Phi}(t) \right| A \left| \overline{\Phi}(t) \right\rangle &= \frac{2\varepsilon G m}{\hbar} \int \mathrm{d}^3r \int \mathrm{d}^3r' \left\langle \overline{\Phi}(t) \right| \left[\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) - D_\Phi(\mathbf{r})\right] A \left| \overline{\Phi}(t) \right\rangle \times \frac{1}{|\mathbf{r}-\mathbf{r}'|}\, n_G(\mathbf{r}') \\ &= \frac{2\varepsilon a G m}{\hbar} \int \mathrm{d}^3r \int \mathrm{d}^3r' \left[\left\langle \overline{\Phi}(t) \right| \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) \left| \overline{\Phi}(t) \right\rangle - D_\Phi(\mathbf{r})\right] \times \frac{1}{|\mathbf{r}-\mathbf{r}'|}\, n_G(\mathbf{r}') = 0\end{aligned} \tag{19}$$

The situation is therefore different from that obtained with GRW and CSL theories [24,25], where the localization mechanism constantly transfers energy to all particles at a small rate: in our model, if the system is in a stationary state, thermal equilibrium for instance, its energy remains constant. The reason for this difference is that, in

GRW and CSL theories, the random localization process involves a noise that is discontinuous in time, and therefore has a very broad spectrum (infinite in the case of a Wiener process); it cannot be treated as a first order perturbation and, for instance, the Ito term has to be included. In our model, the localization term is continuous and has a limited frequency spectrum (determined by the motion of the Bohmian positions); since the coupling constant is very small, it can be treated by first order perturbation theory, and has a much softer effect.

### 2.2 Fast resolution of QSMDS

Assume now that the quantum state describes a QSMDS situation, for instance a measurement pointer (or any macroscopic object) in a superposition of two quantum states localized in two different regions of space. By contrast, the Bohmian positions remain grouped together, forming a cluster that occupies only one of these regions of space. Therefore, in the two branches of the state vector, a strong mismatch then occurs between the quantum density of particles and the Bohmian density (but with a different sign), so that the effect of the localization operator $L$ on these branches is significantly different. To evaluate its consequences we can, in (13), ignore the normalization term in $D_\Phi(\mathbf{r})$, which affects both branches in the same way and does not change their relative amplitude. In the "full component" where the Bohmian density accompanies the quantum density, the localisation term in the right hand side of (10) multiplies the wave function by a number that is of the order of (half of the absolute value of) the self-gravitational energy $E_{\mathrm{sg}}$ of the pointer, multiplied by the constant $\varepsilon$; in the "empty component", it multiplies the wave function by an energy that is negligible with respect to this self-gravitational energy. Altogether, the differential effect takes place with a time constant of the order of:

$$\tau_{\mathrm{collapse}} \simeq \frac{\hbar}{\varepsilon |E_{\mathrm{sg}}|} \tag{20}$$

with:

$$|E_{\mathrm{sg}}| \simeq G\frac{M^2}{L} \tag{21}$$

where $M$ is the mass of the pointer and $L$ its size (we assume that the two wave packets of the pointer are separated by approximately its size, or more). If, for instance, $L = 0.1\,\mathrm{mm}$ and $M = 10^{-6}\,\mathrm{g}$, and assuming $\varepsilon = 10^{-3}$, we find:

$$\tau_{\mathrm{collapse}} \simeq 10^{-6}\,\mathrm{s} \tag{22}$$

We note that $E_{\mathrm{sg}}$ varies as the fifth power of the size of the pointer (at constant density). For instance, if $L = 1\,\mu\mathrm{m}$, we obtain a long collapse time $\tau_{\mathrm{collapse}} \simeq 10^4\,\mathrm{s}$. In experiments such as those of reference [36], very large molecules could fly on different path without being collapsed if the duration of the flight is shorter than this time. The model thus predicts a relatively sharp border between small objects that can reach and stay in a quantum superposition of remote states, and larger ones that almost immediately get projected onto one single location. As discussed in [30], the origin of this projection is the cohesive internal force of solid objects, which forces the Bohmian positions to remain clustered together; gases that do not have this internal cohesion do not undergo the same effect. Interestingly, in the correlated worldline (CWL) theory of quantum gravity [37,38], fifth powers of the masses also appear in the mutual binding energy for paths.

### 2.3 Large systems

In most situations (except, of course, during the appearance of a QSMDS), the space distribution of Bohmian variables accurately coincides with the quantum space distribution $D_\Phi(\mathbf{r})$. Assuming that the gravitational attraction originates from the distribution of Bohmian positions is not very different than assuming that the source of attraction is the quantum distribution $D_\Phi(\mathbf{r})$. The effect of the localization term will then just be to (slowly) localize the macroscopic system inside itself, or to move towards region of lower gravitational potential. This term should have no observable effect, except maybe on very long time and space scales such as those considered in astrophysics; its effect is somewhat reminiscent of the attraction of the so called dark matter.

We note in passing that macroscopic quantum superpositions of states that do not produce different spatial distributions of masses are not reduced by the localization process of the model. For instance, if the flow of electrons in a superconducting ring is in a superposition of two states having rotations in opposite directions, no significant collapse takes place. Fast collapse occurs only to resolve QSMDS involving different gravitational fields, as suggested by Penrose [11].

### 2.4 Measurements

When an apparatus $M$ is used to measure a quantum system $S$, both physical systems become entangled under the effect of their mutual interaction. The state vector then splits into several branches, each containing a state of $S$ that is an eigenstate of the measured observable. During the first stages of measurement, as long as the entanglement remains microscopic, the localization term plays no special role. But, when the entanglement involves states of $M$ involving significantly different distributions of masses in space, for instance different positions of a pointer, then the fast collective collapse takes place: all branches but one of the state vector vanish. The collapse process is therefore initiated inside the measurement apparatus, but immediately propagates back to $S$ by a standard quantum nonlocal effect. This is, for instance, what happens in a Bell experiment. No collapse therefore occurs before a significant part of the measurement apparatus $M$ is part of the entanglement. The result of measurement is determined by the initial Bohmian positions of all particles and, as discussed in [39], in some cases the result is primarily determined by the initial Bohmian positions of the measurement apparatus.

This scenario fits rather well with an old quotation by Pascual Jordan[1]: "observations not only disturb what has

[1] As quoted by Bell in [40].

to be measured, they *produce* it. In a measurement of position, the electron is forced to a decision. We compel it to assume a definite position; previously it was neither here nor there, it had not yet made its decision for a definite position...".

### 2.5 No signaling

When introducing nonlinearities in quantum dynamics, one should be careful about avoiding superluminal communications [42–44]. In the GRW [24] and CSL [25] versions of modified Schrödinger dynamics, nonlinearity and stochasticity compensate each other to cancel superluminal signaling. Similarly, the nonlinear Schrödinger-Newton equation can be made compatible with the no-signaling requirement by changing it to a stochastic differential equation [45]. Here, the situation is somewhat different: the nonlinearity is not introduced as a term coupling the state vector directly to itself, but by the reaction of Bohmian positions onto the wave function; the stochasticity does not arise from a random process constantly acting on the wave function, but from the random values of the initial Bohmian positions.

In order to ensure that the Hamiltonian $H_G(\varepsilon = 0)$ is nonsignaling, we introduce a retarded potential into in equation (2):

$$n_G(\mathbf{r}') \Rightarrow n_G(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}) \tag{23}$$

where $c$ is the speed of light. We then just have to check that the localization term proportional to $\varepsilon$ is also nonsignaling.

Assume that the system, described by the density operator $\rho(t) = |\Psi(t)\rangle \langle\Psi(t)|$, is made of two remote subsystems $A$ and $B$, respectively occupying regions of space $S_A$ and $S_B$, and described by the partial density operators $\rho_A(t)$ and $\rho_B(t)$. We denote $\{|n_A\rangle\}$ an ensemble of states of $A$ providing an orthonormal basis, and $\{|n_B\rangle\}$ a similar basis for system $B$; for instance, $n_A$ and $n_B$ are abbreviated notations for the positions of the $N_A$ particles that are inside $S_A$, and $N_B$ particles inside $S_B$, repectively. The evolution of the matrix elements of $\rho_A(t)$ introduced by the localization term in $\varepsilon$ is given by:

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{\mathrm{loc}} \langle n_A| \rho_A(t) |n'_A\rangle = \frac{2\varepsilon G m}{\hbar} \sum_{n_B} \langle n_A, n_B| \int \mathrm{d}^3 r \int \mathrm{d}^3 r' \left[\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) - D_\Phi(\mathbf{r}), \rho(t)\right]_+ n_G(\mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}) |n'_A, n_B\rangle \tag{24}$$

where $[C, D]_+$ denotes the anticommutator of $C$ and $D$.

We now assume that system $A$ is microscopic, but that $B$ is macroscopic, and that at some time it is driven to a QSMDS, for instance because a quantum measurement is performed in this region $B$. We are interested in the possible effects on the partial density operator $\rho_A(t)$ of the resolution of this QSMDS by the localization operator. The operator in the right hand side of (24) contains the sum of four contributions: $L_{AA}$, $L_{BB}$, $L_{AB}$ and $L_{BA}$. Here the first index $A$ (or $B$) indicates that the integration variable $\mathbf{r}'$ lies in region $S_A$ (or $S_B$), which determines the source of localization; the second index indicates that the integration variable $\mathbf{r}$ lies in region $S_A$ (or $S_B$), which determines the target of the localization process. Since we assume that $A$ is microscopic, we can ignore $L_{AA}$, which is local and remains extremely small since $A$ is microscopic. We are actually only interested in the terms having a macroscopic source, in other words in the effects of $L_{BB}$ and $L_{BA}$.

In fact, $L_{BB}$ is clearly the most important term. It looks local since it corresponds to a localization occurring entirely in region $S_B$ by the collective spontaneous localization process discussed in Section 2.2; nevertheless, since it acts on the density operator $\rho(t)$, which is a nonlocal object if systems $A$ and $B$ are in an entangled state, this term can introduce quantum nonlocality (in particular violations of the Bell inequalities). For instance, if the measurement is performed on two spin 1/2 particles in a singlet spin state, as soon as a single spin measurement is performed in region $S_B$ along a direction $\mathbf{u}$, the localization term will cancel one component of the singlet state; which component is cancelled depends on the result of measurement. In other words, in a single realization of the experiment, the spin state in region $S - A$ will immediately be projected onto the opposite spin state on the same direction $\mathbf{u}$. It is nevertheless well-known that this nonlocatity does not imply any possible superluminal communication – this is the famous "peaceful coexistence between quantum mechanics and relativity" [46]. Indeed, if we consider the average over many realizations, the density matrix of system $A$ remains completely independent of $\mathbf{u}$. Technically, while in the right hand side of (24) $n_G$ fluctuates in region $B$ from one realization to the next, on average it can be replaced by the local density associated with the standard (non-collapsed) solution of the Schrödinger equation; this provides the average effect of the localization on the density operator of $A$. So, term $L_{BB}$ ensures that we recover the usual nonlocal quantum correlations between the remote subsystems $A$ and $B$, the violation of the Bell inequalities, etc., but without any superluminal communication.

We finally have to consider the effect of the term $L_{BA}$. It also implies that the measurement result obtained in region $S_B$ may influence the evolution of the density operator $\rho_A(t)$, but the effect is much weaker that that of $L_{BB}$ since it tends to zero when the distance between regions $S_A$ and $S_B$ increases. Again, the average of this effect over many realizations is obtained by replacing $n_G$ by the standard quantum density in space. It is not signaling because of the delay $|\mathbf{r} - \mathbf{r}'|/c$ appearing in the right hand side of (24): whatever is done to change the Bohmian density inside subsystem $B$ cannot affect the evolution of subsystem $A$ at any time earlier than the minimum delay required by relativity.

For the sake of simplicity, we have assumed that $A$ is microscopic and $B$ macroscopic, but the discussion could easily be generalized to the case where both are macroscopic. Our general conclusion, therefore, is that the model is nonsignaling, at least in all situations that we have considered.

### 2.6 Differences and similarities with GRW/CSL theories

In the dynamical equations of the model, we have assumed that the Bohmian position of every particle is the source of gravity acting on all other particles. This is of course necessary for the Hermitian part of the Hamiltonian (obtained with $\varepsilon = 0$) if one wishes to reproduce the usual effects of gravity. But we have also assumed that this is true for the antihermitian term (term in $\varepsilon$), introducing in this way "mutual collapse terms". As a consequence, our localization term in the dynamical equation is similar to a two-body interaction term. By contrast, the localization term of GRW or CSL theories is rather described by a single-particle potential: the state vector is subjected to the effect of random localization terms acting on all particles independently, with a probability rule that depends on the values of the wave function at the positions of all particles. In other words, in our model the collapse is a collective effect, by contrast with GRW/CSL theories. This difference has several consequences.

A first consequence is that, within our model, the localization rate varies roughly proportionally to the square of the number of particles involved in a QSMDS. Therefore, much smaller values of the collapse coupling constant can be used, without losing a very fast collapse rate of QSMDS. In particular, this explains why the undesirable heating effects initially predicted in [8] with a gravitational collapse do not occur here.

Another consequence is that, as discussed in Section 2.1, the localization process of this model is intrinsically softer than that of GRW and CSL, which have a very short correlation time and therefore a broad spectrum (actually infinitely broad); here, the gravitational attraction towards Bohmian position is continuous in time, so that it can be treated perturbatively to first order (for instance, it does not introduce Ito terms). As discussed in Section 2.3, for large solid bodies, we obtain an effect of localization that is much weaker than that of GRW/CSL theories [30], so that it should be more difficult to detect experimentally (each particle in a solid is localized only inside a large body).

Our model does not require to postulate a probability rule for the random localization field, without any other justification than recovering the Born rule: the correlations between the motions of the Bohmian position, guided as usually by the wave function in the configuration space, are sufficient to ensure a spatial localization of large massive objects, while the constant relaxation towards quantum equilibrium [35] automatically leads to the Born rule.

Pearle and Squires have remarked that the rate of collapse of GRW and CSL theories should be proportional to the mass, indicating a possible relation between collapse and gravitation [47]. To connect our model with these theories, one can modify it by assuming, for instance, that the Bohmian position of each particle is the source of localization for this particle only. The collapse then loses its collective properties and the model becomes more similar to GRW and CSL, maybe even equivalent. If it were equivalent, this would mean that the constant randomness of GRW and CSL theories, contained in their "probability rule", can also be interpreted within a deterministic dynamics in terms of random initial values of the Bohmian positions. We have not explored this question.

### 2.7 EEQT theory

The Event-Enhanced-Quantum-theory (EEQT) [48] proposes a similar method to describe individual quantum systems and to explain why, in a measurement process, "potential properties of a quantum system become actual". It also enhances the standard quantum description of a system by replacing the usual space of states by a family of spaces, labelled with an index $\alpha$, representing the pure state of a classical system $C$. An "event" is defined by a change of the value of $\alpha$. Operators are labelled by two indices $\alpha$ and $\alpha'$, and not necessarily self-adjoint, as the non-Hermitian localization term we have introduced. A back action of the classical system is also introduced. Under these conditions, $\alpha$ plays a role in EEQT theory that is similar to the role of Bohmian positions in our model. The main difference is that the evolution of $\alpha$ is not deterministic, but given by a Markov process.

## 3 Conclusion

We have introduced two basic postulates: the source of gravitation is the Bohmian density of particles, not the quantum density; the gravitational coupling constant includes a small imaginary component. With these two assumptions, predictions that are compatible with presently known facts are obtained, including the appearance of single results in experiments. The dynamics is such that the mathematical objects (wave function and positions) constantly follow the physical observations closely; there is no need to update the value of the wave function in order to include new information. For instance, if a sequence of measurements is performed on the same quantum object, its state vector automatically includes the information obtained in the previous measurements; there is no need to add a state vector reduction by hand, or to keep empty components of the state vector. As discussed in [30] in more detail, the model remains compatible with a whole range of possible interpretations and ontologies.

In this model, the quantum collapse is nothing but a consequence of the internal cohesion of macroscopic objects and of their gravitational self-attraction [30]. The mutual attraction between the particles of the object forces all Bohmian positions to remain grouped together, because they have to occupy regions of the configuration space where the many particle wave function does not vanish, a consequence of standard dBB theory. We then assume that these positions collapse the state vector around them: in equation (13), the source of gravitational attraction is the Bohmian density, instead of the quantum density $D_\Phi(\mathbf{r})$ appearing in the Schrödinger-Newton equation, discussed in detail for instance in [31]. As early as in 1965, Bohm and Bub [49] proposed to introduce a collapse dynamics involving hidden variables (the components of a vector in the dual space of the Hilbert space in their case). As mentioned in the introduction, Penrose [11] suggested in 1996 that, when a QSMDS involving different spatial distribution of masses (and therefore different space-time configurations) creates an energy fluctuation $\Delta E$, the QSMDS spontaneously decays in a time of the order of $\hbar/\Delta E$. The spontaneous collapse arises because of an energy mismatch between two (or more) components of the QSMDS. In this model, the primary origin of the collapse is a mismatch between two densities of space, the quantum density and the Bohmian density; this in turn creates a mismatch of gravitational energy in different components of the QSLMDS and achieves Penrose's scheme, but without any particular general relativistic effect. Recently, Tilloy has proposed a modification of the GRW theory where the sources of a classical gravitational field are the collapse space-time events of that theory [50].

Depending on one's point of view, the role of the Bohmian positions can be seen as more, or less important, than in standard dBB theory. In the dynamics, they certainly play a more active role than in dBB theory, where the positions do not appear in the dynamical equation giving the evolution of the state vector, but just follow the spatial variations of the wave function. Here, the Bohmian positions act as mathematical attractors of the state vector $|\Phi(t)\rangle$ through the gravitational term (including its small dissipative component in $\varepsilon$). This introduces a nonlinearity in the dynamics of $|\Phi(t)\rangle$. Nevertheless, as we have seen, in most situations this change has very little effect on the evolution of $|\Phi(t)\rangle$ – except in situations where QSMDS appear, which are then rapidly projected by this term. The model therefore illustrates how the addition of a single additional variable to the standard equations, namely a point position in the configuration space, allows one to significantly enrich the dynamics and to take into account collapse situations.

From a purely interpretative point of view, one can see this continuous attraction as a pure mathematical ingredient to replace the stochastic fields of GRW and CSL theories, as well as their probability rule. One can then hold a view where the Bohmian positions are just mathematical objects creating this attraction, and where physical reality is directly represented, for instance, by the quantum density $D_\Phi(\mathbf{r})$. But it is also perfectly possible to consider that all the individual Bohmian positions of the particles provide a direct representation of reality, as usual in dBB theory.

This model is in the line of calculations where gravity is treated classically, within general relativity. This remains compatible with a classical structure of space-time, in which the various quantum fields (electromagnetic for instance) propagate (semi-classical gravity [51]); such schemes are sometimes useful in quantum cosmogenesis [53–55]. The circularity of the defintion of time in quantum theory [17] is avoided. A standard approach to semi-classical gravity is to use a quantum average of the energy momentum tensor operator to construct the Einstein tensor [56–58]. Nevertheless, paradoxes may then arise: for instance, if a body is in a quantum superposition of two locations, each localization of the body attracts the other. Also, as discussed by Eppely and Hannah [59], one could in principle measure directly the modulus of the wave function, and therefore obtain superluminal signaling. Other arguments have been built, involving thought interference experiment, to discuss possible inconsistencies, or to plead in favor of a semi-classical theory of gravitation [60–64]. In our model, as in that of reference [50], the paradoxes arising from of delocalized sources of gravity disappear: in each realization of an experiment, the source of gravitation always remains localized in space (since it originates from the Bohmian positions). Of course, in most situations (when no QSMDS occurs) it is practically equivalent to take the Bohmian positions, or the average quantum density of particles, as the source of gravity, due to the quantum equilibrium conditions. In this sense, the predictions of this model are very similar to those of the theory of semiclassical gravity proposed by Tilloy and Diosi [13], the major difference being that their approach is based on a stochastic spontaneous localization, while no random perturbation is invoked in the present article.

At this stage, the model remains very elementary, in particular because its treatment of gravity remains simply Newtonian, not Einsteinian: for instance, it does not include gravitational waves. The hope is that the model could be an approximation of some more elaborate theory, compatible with general relativity. One could also speculate about a possible generalization to a quantum treatment of a gravitational field, still having its sources in the Bohmian positions of the particles. One hope could be to find a justification of the complex value of the coupling constant by analogy with electromagnetic spontaneous emission, also taking into account the intrinsic nonlinear character of general relativity. This, of course, remains completely speculative. As it is, the model is definitely in the line of a semi-classical treatment of gravity.

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