Quantum Fluctuations Hinder Finite-Time Information Erasure near the Landauer Limit

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Information is physical but information is also processed in finite time. Where computing protocols are concerned, finite-time processing in the quantum regime can dynamically generate coherence. Here we show that this can have significant thermodynamic implications. We demonstrate that quantum coherence generated in the energy eigenbasis of a system undergoing a finite-time information erasure protocol yields rare events with extreme dissipation. These fluctuations are of purely quantum origin. By studying the full statistics of the dissipated heat in the slow-driving limit, we prove that coherence provides a non-negative contribution to all statistical cumulants. Using the simple and paradigmatic example of single bit erasure, we show that these extreme dissipation events yield distinct, experimentally distinguishable signatures.

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Landauer's principle states that any logically irreversible computation produces entropy, which dissipates heat to noninformation bearing degrees of freedom [1]. This basic principle not only sets an ultimate physical limit to information processing but also forms the foundation of the thermodynamics of computation [2,3] and information [4–8], while playing a pivotal role in the resolution of the Maxwell demon paradox [9,10]. The most elementary logically irreversible process is the erasure of one bit of information, which dissipates an amount of heat $q \geq k_B T \ln(2)$ to the environment, where k_B is Boltzmann's constant and T is the temperature. This fundamental lower bound on dissipated heat is known as the Landauer limit.

In reality, any physical implementation of information erasure takes place under nonequilibrium conditions, where a possibly microscopic system (information bearing degree of freedom) is manipulated in finite time while in contact with a heat bath. In this setting, fluctuations become significant, and path-dependent thermodynamic quantities, such as heat and work, are described by probability distributions [11–16]. This has important consequences for heat management in nanoscale devices, which must be designed to tolerate large and potentially destructive fluctuations. As information processing technology encroaches on the small scale where quantum effects take hold, it thus becomes crucial to understand how quantum as well as thermal fluctuations contribute to dissipation during the erasure process.

Minimizing dissipation typically requires slow driving in order to remain in the quasistatic regime. This has been highlighted by the first generation of experiments aiming to experimentally study information erasure near the Landauer limit on both classical [17–22] and quantum [23–26] platforms. In particular, the probability distributions of work and heat during a finite-time protocol were extracted in pioneering experiments on Brownian particles confined by tunable double-well potentials [18,19]. In the quasistatic regime, it was found that the dissipated heat approaches the Landauer limit on average [18], while its probability distribution becomes Gaussian [19] with a variance constrained by the work fluctuation-dissipation relation [27]. However, experiments exploring the full heat statistics of quasistatic erasure have so far been limited to a classical regime, leaving open the question of how quantum effects influence the heat distribution.

Here we demonstrate that quantum coherence is always detrimental for the attainability of the fundamental Landauer limit during slow erasure protocols. More precisely, we prove that quantum coherence generated in the energy eigenbasis of a slowly driven system yields a non-negative contribution to all statistical cumulants of the dissipated heat and renders the associated probability distribution non-Gaussian. Coherent control therefore increases the overall likelihood of dissipation above the Landauer bound due to the heat distribution developing a significant skewness. We exemplify this general principle by studying the erasure of one bit of information stored in a quantum two-level system, as illustrated schematically in Fig. 1. We find that quantum fluctuations generate distinct and, in principle, experimentally distinguishable signatures in the heat statistics, consisting of rare events with extreme dissipation $q \gg k_B T$. Despite their rarity, the significance of such processes is clear in light of the many billions of bits that are irreversibly processed each second in modern computer hardware. Aside from unambiguously demonstrating a quantum effect in information thermodynamics, our findings imply that control strategies designed to suppress quantum fluctuations may be necessary to mitigate dissipation in miniaturized information processors, in agreement with results from single-shot statistical mechanics [28].

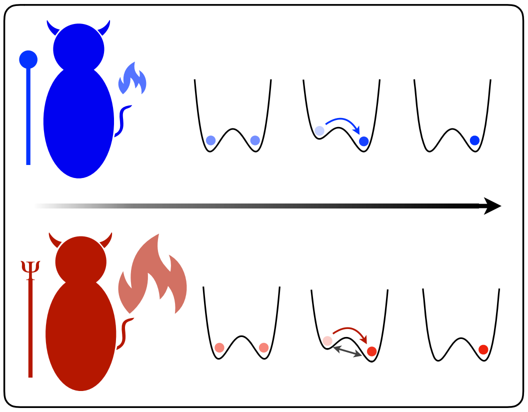

FIG. 1. Schematic of the erasure protocol and our main results. An intelligent being (demon) performs erasure through a controlled process that resets a physical bit of information to a fixed reference state. In this example, the bit is encoded in the position of a particle confined by a double-well potential. Classically, erasure is performed by raising the potential of one well until thermal fluctuations drive the particle into the lower-energy state, at the cost of dissipating some heat into the environment. Quantum mechanics allows the particle to coherently tunnel under the barrier as well as hop over it, leading to large quantum fluctuations in the dissipated heat.

Erasure protocol.—We note that Landauer's principle for *finite* quantum baths [29–33] has recently been experimentally explored in Refs. [23,24]. In this work, we consider an erasure protocol where a controllable quantum system with encoded information is continuously connected to noninformation bearing degrees of freedom modeled as an *infinite* heat bath. Specifically, information encoded in a quantum system of finite dimension d, described by a maximally mixed state $\hat{\mathbb{I}}/d$, is erased by bringing the system to its ground state $|0\rangle\langle 0|$, resulting in a decrease in information entropy $\Delta S = -\log d$. This is achieved by slowly varying a control Hamiltonian $\hat{H}_t$ over a finite-time interval $t \in [0, \tau]$ while the system is weakly coupled to a thermal reservoir at inverse temperature $\beta = 1/k_B T$. We assume Markovian dynamics generated by an adiabatic Lindblad equation [34], $\dot{\hat{\rho}}_t = \mathcal{L}_t(\hat{\rho}_t)$, where the generator $\mathcal{L}_t$ obeys quantum detailed balance with respect to the Hamiltonian $\hat{H}_t$ at all times [35]. This condition ensures a thermal instantaneous fixed point, $\mathcal{L}_t(\hat{\pi}_t) = 0$, where $\hat{\pi}_t = e^{-\beta \hat{H}_t}/\mathrm{Tr}(e^{-\beta \hat{H}_t})$. Erasure can be realized by first taking an initial Hamiltonian with $\hat{H}_0 \simeq 0$ relative to the thermal energy $k_B T$, then increasing its energy gaps until they far exceed $k_B T$. If one assumes that the system is in equilibrium at the end of the process, this results in effective boundary conditions $\hat{\rho}_0 = \hat{\pi}_0 \simeq \hat{\mathbb{I}}/d$ and $\hat{\rho}_\tau = \hat{\pi}_\tau \simeq |0\rangle\langle 0|$.

Heat statistics.—Having introduced our erasure protocol, we now discuss the full counting statistics of the dissipated heat. In the weak-coupling limit, heat is unambiguously identified with the change in energy of the reservoir [36]. For Lindblad dynamics with detailed balance, the evolution may be unraveled into quantum-jump trajectories [37], where heat exchange is associated with the emission and absorption of energy quanta by the driven quantum system [38–41]. Operationally, each trajectory represents an individual run of an experiment in which the environment is continuously monitored by direct detection of the emitted and absorbed quanta [42]. This is formally described by a set of coarse-grained time points at which measurements occur, $\tau \geq t_N \geq \cdots \geq t_1 \geq t_0 = 0$, separated by an increment δt much smaller than the characteristic timescale of dissipation. The system evolution from time $t_n \to t_{n+1}$ is given by the quantum channel $\mathcal{T}_n \coloneqq e^{\delta t \mathcal{L}_{t_n}} = \sum_{x_n} \mathcal{T}_{x_n}$, where $\mathcal{T}_{x_n}(\cdot) = \hat{K}_{x_n}(t_n)(\cdot)\hat{K}^\dagger_{x_n}(t_n)$ form a set of Kraus operators satisfying $\sum_{x_n} \hat{K}^\dagger_{x_n}(t_n)\hat{K}_{x_n}(t_n) = \hat{\mathbb{I}}$ and x_n labels the distinguishable outputs of the detector. Each trajectory of the open system is then specified by its measurement record, i.e., a sequence of the form $\Gamma \coloneqq \{x_0, \ldots, x_N\}$ occurring with probability

$$p(\Gamma) \coloneqq \frac{1}{d}\langle 0|\prod_{n=0}^{N}\mathcal{T}_{x_n}(\hat{\mathbb{I}})|0\rangle. \tag{1}$$

To ensure detailed balance [40,43], the Kraus operators are taken to satisfy $[\hat{H}_t, \hat{K}_x(t)] = -\hbar\omega_x(t)\hat{K}_x(t)$, where $\hbar\omega_x(t)$ are differences between the eigenvalues of $\hat{H}_t$. Thus, $\omega_x > 0$ $(\omega_x < 0)$ represents a detected emission (absorption) while $\omega_x = 0$ represents no detection. This assumption ensures that heat entering the environment may be identified along each trajectory Γ, being given by the sum of these energy changes:

$$q(\Gamma) \coloneqq -\sum_{n=0}^{N}\hbar\omega_{x_n}(t_n). \tag{2}$$

We note that in the weak-coupling regime this is equivalent to the outcome of a two-point measurement of the environment's energy at the beginning and end of the protocol [44–46]. The average heat flux is given by $\langle \dot{q} \rangle = \mathrm{Tr}(\hat{H}_t \dot{\hat{\rho}}_t)$, consistent with well-known results for weak-coupling Lindblad dynamics [47].

It is convenient to define the excess stochastic heat,

$$\tilde{q}(\Gamma) \coloneqq q(\Gamma) - k_B T \log d, \tag{3}$$

which quantifies the additional heat in excess of the Landauer bound. The full statistics of excess heat can be obtained from the cumulant generating function (CGF), evaluated in the continuum limit $\delta t \to 0$:

$$\mathcal{K}_q(u) \coloneqq \ln \sum_{\{\Gamma\}} e^{-u\tilde{q}(\Gamma)} p(\Gamma). \tag{4}$$

This provides the cumulants according to $\kappa_k = (-1)^k \frac{d^k}{du^k}\mathcal{K}_q(u)|_{u=0}$, where $\kappa_1 = \langle q \rangle - k_B T \log d$ is the average excess heat, $\kappa_2 = \mathrm{Var}(q) = \langle q^2 \rangle - \langle q \rangle^2$ is the variance, and so forth.

Role of coherence in erasure.—We now come to the main finding of our work, namely that quantum coherence generates additional dissipation during information erasure. We focus on protocols close to the quasistatic limit, where the dissipation approaches the Landauer bound. This requires the Hamiltonian to be driven slowly relative to the relaxation timescale of the dynamics, implying that the system state remains close to equilibrium at all times. We may therefore use an expansion of the form $\hat{\rho}_t \simeq \hat{\pi}_t + \tau^{-1}\delta\hat{\rho}_t$, with $\delta\hat{\rho}_t$ a linear-order perturbation to the equilibrium state $\hat{\pi}_t$.

Neglecting corrections of order $\mathcal{O}(\tau^{-2})$, we find that the full statistics of excess heat in the slow-driving limit can be separated into a classical and quantum part (see Supplemental Material [48]):

$$\mathcal{K}_q(u) = \mathcal{K}_q^d(u) + \mathcal{K}_q^c(u). \tag{5}$$

Because of the additivity of the CGFs we may interpret the total excess heat as a sum of two independent random variables, $\tilde{q}(\Gamma) = \tilde{q}_d(\Gamma) + \tilde{q}_c(\Gamma)$, with $\tilde{q}_d(\Gamma)$ described by a classical (diagonal) CGF $\mathcal{K}_q^d(u)$ and cumulants of $\tilde{q}_c(\Gamma)$ given by the quantum (coherent) CGF $\mathcal{K}_q^c(u)$. These different contributions to the heat statistics relate directly to the different ways a quantum state can evolve, through changes to either the populations or the coherences in the energy eigenbasis [48]. Specifically, the classical CGF is given by

$$\mathcal{K}_q^d(u) = k_B T(u - k_B T u^2) \int_0^\tau dt \frac{\partial}{\partial t} S(\mathcal{D}_{\hat{H}_s}(\hat{\rho}_t) \| \hat{\pi}_s)|_{s=t}, \tag{6}$$

where $S(\hat{\rho}\|\hat{\sigma}) = \mathrm{Tr}(\hat{\rho}\ln(\hat{\rho})) - \mathrm{Tr}(\hat{\rho}\ln(\hat{\sigma}))$ is the quantum relative entropy and $\mathcal{D}_{\hat{H}_t}(\cdot) = \sum_n |n_t\rangle\langle n_t|(\cdot)|n_t\rangle\langle n_t|$ denotes the dephasing map in the *instantaneous* energy eigenbasis $\{|n_t\rangle\}$ of $\hat{H}_t$. Equation (6) expresses the fact that classical contributions to the excess heat occur when the system populations deviate from the instantaneous Boltzmann distribution. Furthermore, the quantum CGF can be identified as

$$\mathcal{K}_q^c(u) = -u k_B T \int_0^\tau dt \frac{\partial}{\partial t} S_{1-uk_BT}(\hat{\rho}_t \| \mathcal{D}_{\hat{H}_s}(\hat{\rho}_t))|_{s=t}, \tag{7}$$

where $S_\alpha(\hat{\rho}\|\hat{\sigma}) = (\alpha - 1)^{-1} \ln \mathrm{Tr}(\hat{\rho}^\alpha \hat{\sigma}^{1-\alpha})$ for $\alpha \in (0,1) \cup (1,\infty)$ represents the quantum Renyi divergence. The function $S_\alpha(\hat{\rho}_t \| \mathcal{D}_{\hat{H}_t}(\hat{\rho}_t))$ can be interpreted as a proper measure of asymmetry with respect to the instantaneous energy eigenbasis [58], which is closely related to the amount of coherence contained in the state [59]. The first cumulant of Eq. (7) is proportional to the relative entropy of coherence [60], which has previously been identified as a quantum contribution to average entropy production in open [61–63] and closed [64] systems. A similar division into classical and quantum components was obtained in Ref. [65] for work statistics in the slow-driving limit.

Remarkably, the splitting embodied by Eq. (5) puts constraints of nonclassical origin on the full statistics of dissipated heat. To see this, let us first convert the diagonal part $\mathcal{K}_q^d(u)$ into a probability distribution via an inverse Laplace transform. This yields a Gaussian distribution with mean and variance connected by $\langle \tilde{q}_d \rangle = \frac{1}{2}\beta \mathrm{Var}(\tilde{q}_d)$, as expected for a classical process in the slow-driving limit [66]. It follows that the classical heat distribution obeys the Landauer bound, $\langle \tilde{q}_d \rangle \geq 0$. Turning to the quantum contribution, no such straightforward expression can be obtained for the distribution $P(\tilde{q}_c)$ due to the complicated dependence of the quantum covariance (7) on the counting field u. Despite this, one may prove that the cumulants of $\tilde{q}_c$ are all monotonically nondecreasing in time [48]:

$$(-1)^k \frac{d^k}{du^k} \dot{\mathcal{K}}_q^c(u)|_{u=0} \geq 0, \quad \forall \ k. \tag{8}$$

This immediately implies that coherence imparts a non-negative contribution to the mean heat dissipated during erasure, i.e.,

$$\langle q \rangle = k_B T \ln(d) + \langle \tilde{q}_d \rangle + \langle \tilde{q}_c \rangle, \quad \text{with} \quad \langle \tilde{q}_c \rangle \geq 0. \tag{9}$$

Furthermore, all higher cumulants are also non-negative, implying increased fluctuations that will generally exhibit positive skew and kurtosis. As a consequence, the overall heat distribution can be highly non-Gaussian, in stark contrast to the classical case.

These results have profound repercussions for the erasure of information stored in a quantum system. Manipulating such a system in finite time typically generates coherence due to the presence of several non-commuting terms in the Hamiltonian, a feature which is unavoidable for certain physical architectures. Not only does this lead to a greater energetic cost on average, it also increases the probability of large fluctuations where a quantity of heat $q \gg k_B T \ln(d)$ well above the Landauer bound is dissipated into the surroundings.

Example: Qubit erasure.—To illustrate our findings, we consider an elementary example of erasure where information is stored in a quantum two-level system described by the Hamiltonian

$$\hat{H}_t = \frac{\varepsilon_t}{2}(\cos\theta_t \hat{\sigma}_z + \sin\theta_t \hat{\sigma}_x). \tag{10}$$

This generic Hamiltonian describes the low-energy dynamics of a particle in a double-well potential [67] or a genuinely discrete information storage device such as a charge or spin qubit [68]. Thermal dissipation is modeled by a bosonic heat bath described by an adiabatic Lindblad master equation [34] in the limit of slow driving and weak coupling [48]. Stored information is erased by increasing

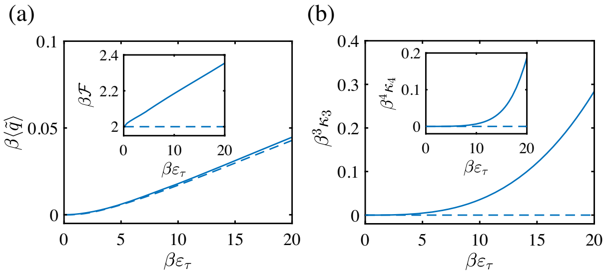

FIG. 2. Heat statistics of slow-driving processes. (a) Mean excess heat (main) and Fano factor, $\mathcal{F}$ (inset). (b) Third cumulant (main) and fourth cumulant (inset) of the heat distribution, demonstrating non-Gaussian statistics. Solid lines show a quantum protocol with $\varepsilon_t = \varepsilon_0 + (\varepsilon_\tau - \varepsilon_0)\sin^2(\pi t/2\tau)$ and $\theta_t = \pi(t/\tau - 1)$, dashed lines show the corresponding classical protocol with identical ε_t but $\theta_t = 0$. The initial energy splitting is $\varepsilon_0 = 0.02\varepsilon_\tau$ and the protocol duration is $\tau = 250/\bar{\gamma}$, where $\bar{\gamma}$ is a characteristic thermalization rate given by the time average of $\gamma_t = \frac{1}{2}\hbar^{-1}\alpha\varepsilon_t\coth(\beta\varepsilon_t/2)$, with $\alpha = 0.191$ the coupling to an Ohmic bath.

the energy splitting ε_t from its initial value, $\varepsilon_0 \approx 0$, to a final value, $\varepsilon_\tau \gg k_B T$, leaving the qubit in its ground state with near-unit probability. The mixing angle θ_t encapsulates the competition between energetic bias ($\hat{\sigma}_z$) and coherent tunneling ($\hat{\sigma}_x$). If θ_t is constant, Eq. (10) describes a classical bit. Conversely, when $\dot{\theta}_t \neq 0$—which will generally be the case, e.g., for quantum double-well systems—the protocol is noncommuting.

In Fig. 2 we plot the first four cumulants of the heat distribution, comparing a quantum protocol to a classical process with identical ε_t but $\dot{\theta}_t = 0$. These analytical results are derived in the slow-driving limit at order $\mathcal{O}(\tau^{-1})$ [48]. We show in Fig. 2(a) that the mean excess heat [Eq. (3)] takes small but nonzero values in the erasure regime, $\varepsilon_\tau \gg k_B T$, reflecting the entropy produced in this finite-time process. While the quantum and classical protocols show similar dissipation on average, they differ significantly in their fluctuations. The inset of Fig. 2 shows the Fano factor, $\mathcal{F} = \mathrm{Var}(\tilde{q})/\langle\tilde{q}\rangle$, which is increased by quantum fluctuations above the classical value, $\mathcal{F} = 2k_B T + \mathcal{O}(\tau^{-2})$, that follows from the fluctuation-dissipation relation. The most significant difference arises in higher-order statistics: the nonclassical nature of the heat distribution is witnessed by its third and fourth cumulants, shown in Fig. 2(b). These imply significant skewness and kurtosis and signal the presence of non-Gaussian tails in the distribution.

To reveal the microscopic origin of these tails, we simulate individual runs of the erasure protocol using the quantum-jump trajectory approach [48]. A trajectory is described by a pure state $|\psi_t\rangle$ undergoing continuous time evolution interspersed by stochastic jumps, $|\psi_t\rangle \to |\pm\varepsilon_t\rangle$, where $\hat{H}_t|\pm\varepsilon_t\rangle = \pm\frac{1}{2}\varepsilon_t|\pm\varepsilon_t\rangle$. Each jump transfers a quantum of energy $\hbar\omega = \mp\varepsilon_t$ to the environment. The main panel of Fig. 3 shows the heat distribution obtained by numerically sampling many such trajectories for a quantum process. While the bulk of the distribution is centered around the Landauer bound, we find a few rare trajectories featuring a very large heat transfer, which are associated with nonadiabatic transitions occurring during the driving protocol. For example, consider an emission at some time, which leaves the system in its instantaneous ground state. As the eigenbasis of the Hamiltonian rotates, the state at some later time comprises a superposition of both energy eigenstates. The finite population of the excited state thus opens the possibility for a second emission to occur, potentially leading to massive overall heat transfer. An example of such a trajectory is shown in the inset of Fig. 3. On the contrary, during a classical protocol the state adiabatically follows the Hamiltonian eigenstates between jumps. This implies that an emission can only be followed by an absorption and vice versa, such that the contributions of these alternating events to the heat statistics largely cancel. We note that, apart from these rare trajectories, the heat distributions sampled from quantum protocols are very similar to their classical counterparts, with the bulk of the distribution approaching a Gaussian form as τ increases [48]. The excess skewness and kurtosis of the quantum heat distributions can therefore be attributed entirely to rare, nonadiabatic processes such as the one illustrated in Fig. 3.

Even though such events are statistical outliers, they may have severe consequences for nanoscale heat management. For the data shown in Fig. 3, roughly one trajectory in every thousand involves a nonadiabatic transition. However, the maximum heat dissipated in a single trajectory is more than 30 times larger than the Landauer limit, whereas for the

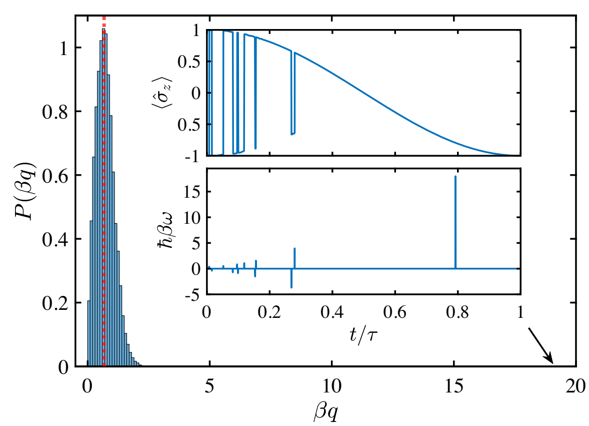

FIG. 3. Quantum-jump trajectory simulation of the coherent qubit erasure protocol of Fig. 2, with $\beta\varepsilon_\tau = 20$. Main panel: heat distribution over 3×10^4 trajectories, with the Landauer bound $\beta q = -\Delta S$ shown by the red dotted line. Inset: dynamics of a rare trajectory with large heat transfer, $q = 19.1k_B T$ (black arrow). Stochastic jumps in the otherwise continuous evolution of $\langle\hat{\sigma}_z\rangle$ (upper inset) are associated with the emission of energy quanta $\hbar\omega = \pm\varepsilon_t$ to the environment (lower inset). Nonadiabatic quantum evolution allows for two consecutive emissions and consequently extreme dissipation.

analogous classical protocol it is less than 4 times larger. This illustrates that quantum coherence drastically increases the probability of extreme heat fluctuations during the process of information erasure. Such events could damage or disrupt small-scale quantum hardware with a low threshold of tolerance for heat dissipation. These are truly quantum fluctuations, in the sense that uncertainty in the transferred heat is increased by the existence of a coherent superposition state of the system together with the quantization of energy exchanged with the environment. In the context of qubit erasure, these quantum fluctuations are experimentally distinguishable from thermal fluctuations since only the former involve consecutive emission or absorption events.

The results presented here can be applied to other logic operations implemented on physical hardware. Indeed, we expect that unique energetic fingerprints may also be discovered in other control protocols that process information in the quantum regime. Fast protocols that push the system far from equilibrium [69] are especially important for computing at high clock speed but are also expected to incur even greater heat fluctuations. Recently developed methods to describe dissipation in driven open quantum systems [70–73] could be used to address this problem in future work.

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