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High-precision determination of the K_{e3} radiative corrections

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A B S T R A C T

We report a high-precision calculation of the Standard Model electroweak radiative corrections in the $K \to \pi e^+ \nu(\gamma)$ decay as a part of the combined theory effort to understand the existing anomaly in the determinations of V_{us}. Our new analysis features a chiral resummation of the large infrared-singular terms in the radiative corrections and a well-under-control strong interaction uncertainty based on the most recent lattice QCD inputs. While being consistent with the current state-of-the-art results obtained from chiral perturbation theory, we reduce the existing theory uncertainty from 10^{-3} to 10^{-4}. Our result suggests that the Standard Model electroweak effects cannot account for the V_{us} anomaly.

An interesting anomaly has recently been observed in V_{us}, which is a top-row element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] in the Standard Model (SM) of particle physics. The measured values of this matrix element stem from two different channels of kaon decay, $K \to \mu\nu(\gamma)$ ($K_{\mu 2}$) and $K \to \pi l^+ \nu(\gamma)$ (K_{l3}), and show a disagreement at the $\sim 3\sigma$ level [3]:

$$\begin{aligned} |V_{us}| &= 0.2252(5)\ (K_{\mu 2})\ , \\ &= 0.2231(7)\ (K_{l3})\ , \end{aligned} \tag{1}$$

which may hint to the existence of physics beyond the Standard Model (BSM). The value obtained from the K_{l3} decay is particularly interesting because it also leads to a violation of the top-row CKM unitarity at $(3-5)\sigma$ upon combining with the most recent updates of V_{ud} [4–7], depending on the amount of nuclear uncertainties assigned to the latter [8,9]. However, despite of an active discussion about the possible BSM origin of the $K_{\mu 2}$–K_{l3} discrepancy [10–18], the current significance level is not yet sufficient to claim a discovery. One of the main obstacles is the large hadronic uncertainty in the electroweak radiative corrections (EWRC), which are the focus of this work.

Among the many studies of the EWRC in K_{l3} [19–29], the standard inputs in global analyses [30,31] are based on chiral perturbation theory (ChPT) which is the low-energy effective field theory of Quantum Chromodynamics (QCD). Within this framework, the "short-distance" electroweak corrections are isolated as a constant factor, while the "long-distance" electromagnetic corrections are calculated up to $\mathcal{O}(e^2p^2)$ [32–34], with e the electric charge and p a small momentum/meson mass. The estimated theory uncertainties in these calculations are of the order 10^{-3}, and originate from: (1) the neglected contributions at $\mathcal{O}(e^2p^4)$, and (2) the contributions from non-perturbative QCD at the chiral symmetry breaking scale $\Lambda_\chi \simeq 4\pi F_\pi$ that exhibit themselves as the poorly-constrained low-energy constants (LECs) in the theory [35,36]. These natural limitations prohibit further improvements of the precision level within the original framework.

In this letter we report a new calculation of the EWRC in K_{e3}. Based on a newly-proposed computational framework [37,38] that hybridizes the classical approach by Sirlin [39] and modern ChPT, we effectively resum the numerically largest terms in the EWRC to all orders in the chiral expansion and significantly reduce the $\mathcal{O}(e^2p^4)$ uncertainty. Also, we utilize the high-precision lattice QCD calculations of the forward axial γW-box diagrams [40,41] to constrain the physics from the non-perturbative QCD. With these improvements, we reduce the theory uncertainty in the EWRC to K_{e3} to an unprecedented level of 10^{-4}. We will outline here the most important steps that lead to the final results, while the full detail of the calculation will appear in a longer paper [42].

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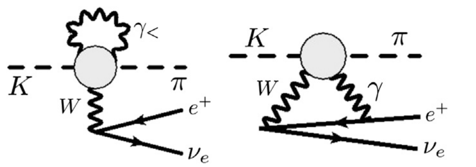

Fig. 1. Non-trivial loop diagrams in the K_{e3} EWRC. A factor $M_W^2/(M_W^2-q'^2)$ is attached to the propagator of $\gamma_<$.

Our primary goal is to study the fractional correction to the K_{e3} decay rate due to EWRC:

$$\delta_{K_{e3}} = \delta\Gamma_{K_{e3}}/\left(\Gamma_{K_{e3}}\right)_{\text{tree}} \tag{2}$$

up to the precision level of 10^{-4}. The denominator in Eq. (2) comes from the tree-level amplitude for $K(p)\to\pi(p')e^+(p_e)\nu(p_\nu)$:

$$M_0 = -\sqrt{2}G_F\bar{u}_{\nu L}\gamma_\lambda v_{eL}F^\lambda(p',p)\ , \tag{3}$$

where G_F is the Fermi constant, $F^\lambda(p',p) = V_{us}^*\left[f_+(t)(p+p')^\lambda + f_-(t)(p-p')^\lambda\right]$ is the charged weak matrix element and $f_\pm(t)$ are the charged weak form factors, with $t=(p-p')^2$. We restrict ourselves to K_{e3} for which the contribution from f_- to the decay rate is suppressed by $m_e^2/M_K^2 \approx 10^{-6}$ and can be neglected.

The full EWRC includes both the virtual corrections and the bremsstrahlung contributions, and we shall start with the former. A generic one-loop correction to the decay amplitude reads:

$$\delta M_{\text{vir}} = -\sqrt{2}G_F\bar{u}_{\nu L}\gamma_\lambda v_{eL}I^\lambda\ , \tag{4}$$

where the loop integrals are contained in I^λ. It results in a shift of the form factors: $f_\pm \to f_\pm + \delta f_\pm$, except that $\delta f_\pm$ can also depend on $s=(p'+p_e)^2$ or $u=(p-p_e)^2$. Again, in K_{e3} only δf_+ is relevant.

We follow the categorization of the different components of the $\mathcal{O}(G_F\alpha)$ virtual corrections in Refs. [37,38], with $\alpha = e^2/4\pi$. First, there are pieces in which the loop integrals are independent of the hadron properties and can be computed analytically. They are contained in Eqs. (2.4) and (2.13) in Ref. [38], which combine to give:

$$(\delta f_+)_{\text{I}} = \left\{\frac{\alpha}{2\pi}\left[\ln\frac{M_Z^2}{m_e^2} - \frac{1}{4}\ln\frac{M_W^2}{m_e^2} + \frac{1}{2}\ln\frac{m_e^2}{M_\gamma^2} - \frac{3}{8} + \frac{1}{2}\tilde{a}_g\right] + \frac{1}{2}\delta_{\text{HO}}^{\text{QED}}\right\} f_+(t)\ , \tag{5}$$

where $\tilde{a}_g = -0.083$ and $\delta_{\text{HO}}^{\text{QED}} = 0.0010(3)$ come from perturbative QCD corrections and the resummation of large QED logarithms, respectively. Notice also that we have introduced a small photon mass M_γ to regularize the infrared (IR)-divergence.

The remaining loop diagrams in the EWRC, in which the entire dependence on hadronic structure is contained, are depicted in Fig. 1. They depend on the following quantities:

$$T^{\mu\nu} \equiv \int d^4x e^{iq'\cdot x}\langle\pi(p')|T\{J_{\text{em}}^\mu(x)J_W^{\nu\dagger}(0)\}|K(p)\rangle$$
$$\Gamma^\mu \equiv \int d^4x e^{iq'\cdot x}\langle\pi(p')|T\{J_{\text{em}}^\mu(x)\partial\cdot J_W^\dagger(0)\}|K(p)\rangle\ , \tag{6}$$

which are both functions of the momenta $\{q',p',p\}$. In particular, we may split the tensor $T^{\mu\nu}$ into two pieces: $T^{\mu\nu} = \left(T^{\mu\nu}\right)_V + \left(T^{\mu\nu}\right)_A$ that contain the vector and axial component of the charged weak current, respectively. With these, the first relevant integral can be written as:

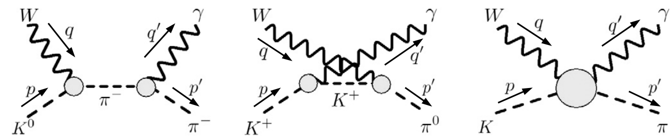

Fig. 2. Pole (left, middle) and seagull diagrams.

$$I_{\mathfrak{A}}^\lambda = -e^2\int\frac{d^4q'}{(2\pi)^4}\frac{1}{\left[(p_e-q')^2-m_e^2\right]\left[q'^2-M_\gamma^2\right]}$$
$$\times\left\{\frac{2p_e\cdot q'q'^\lambda}{q'^2-M_\gamma^2}T_\mu^\mu + 2p_{e\mu}T^{\mu\lambda} - (p-p')_\mu T^{\lambda\mu} + i\Gamma^\lambda\right.$$
$$\left.-i\epsilon^{\mu\nu\alpha\lambda}q'_\alpha\left(T_{\mu\nu}\right)_V\right\}\ , \tag{7}$$

where the first two lines come from Eq. (2.13) of Ref. [38], and the third line is a part of $\delta M_{\gamma W}^A$ in Eq. (2.10) of the same paper.

The operator product expansion (OPE) shows that the $|q'| > \Lambda_\chi$ region does not contribute to the integral $I_{\mathfrak{A}}^\lambda$, therefore only the low-energy expressions of $T^{\mu\nu}$ and Γ^μ are needed. To this end, we find it useful to split them into the "pole" and "seagull" terms respectively, as depicted in Fig. 2:

$$T^{\mu\nu} = T_{\text{pole}}^{\mu\nu} + T_{\text{sg}}^{\mu\nu}\ ,\ \ \Gamma^\mu = \Gamma_{\text{pole}}^\mu + \Gamma_{\text{sg}}^\mu\ . \tag{8}$$

Furthermore, we can obtain the so-called "convection term" by setting $q'\to 0$ in both the electromagnetic form factor and the charged weak vertex of the pole term [43]. It represents the minimal expression that satisfies the exact electromagnetic Ward identity, and thus gives the full IR-divergent structures in the loop integrals.

The seagull term receives contributions from resonances and the many-particle continuum. An estimate operating with low-lying resonances [44–46] suggests that its contribution to $\delta_{K_{e3}}$ is at most 10^{-4}. Note that t-channel exchanges that still retain some sensitivity to the long-range effects do not contribute to Eq. (7). To stay on the conservative side, we assign to it a generic uncertainty of 2×10^{-4}. Therefore, δf_+ derived from $I_{\mathfrak{A}}^\lambda$ is dominated by the pole contribution which is fully determined by the K and π electromagnetic and charged weak form factors. The result splits into two pieces:

$$(\delta f_+)_{\mathfrak{A}} = (\delta f_+)_{\text{II}} + (\delta f_+)_{\mathfrak{A}}^{\text{fin}}\ , \tag{9}$$

where $(\delta f_+)_{\text{II}}$ is a model-independent IR-divergent piece. The IR-finite piece, $(\delta f_+)_{\mathfrak{A}}^{\text{fin}}$, on the other hand, is evaluated numerically by adopting a monopole parameterization of the hadronic form factors [47–49]. Notice that the integral $I_{\mathfrak{A}}^\lambda$ only probes the region $q'\sim p_e\sim p-p'\sim M_K-M_\pi$, where different parameterizations of the form factors are practically indistinguishable. In particular, we find that the main source of the uncertainty is the K^+ mean-square charge radius and the experimental uncertainty thereof, $\langle r_K^2\rangle = 0.34(5)$ fm^2 [47].

The second relevant integral is:

$$I_{\mathfrak{B}}^\lambda = ie^2\int\frac{d^4q'}{(2\pi)^4}\frac{M_W^2}{M_W^2-q'^2}\frac{\epsilon^{\mu\nu\alpha\lambda}q'_\alpha\left(T_{\mu\nu}\right)_A}{\left[(p_e-q')^2-m_e^2\right]q'^2}\ , \tag{10}$$

which picks up the remaining part of $\delta M_{\gamma W}^A$ in Eq. (2.10) of Ref. [38]. It is IR-finite, but probes the physics from $|q'|=0$ all the way up to $|q'|\sim M_W$. A significant amount of theoretical uncertainty thus resides in the region $|q'|\sim\Lambda_\chi$ where non-perturbative QCD takes place, and has been an unsettled issue for decades. The situation is changed following the recent lattice QCD calculations of the so-called "forward axial γW-box":

$$\Box^{VA}_{\gamma W}(\phi_i,\phi_f,M)\equiv\frac{ie^2}{2M^2}\int\frac{d^4q'}{(2\pi)^4}\frac{M_W^2}{M_W^2-q'^2}\frac{\epsilon^{\mu\nu\alpha\beta}q'_\alpha p_\beta}{(q'^2)^2}\times\frac{T^{if}_{\mu\nu}(q',p,p)}{F_+^{if}(0)}\,, \tag{11}$$

where $T^{if}_{\mu\nu}$ is just $T_{\mu\nu}$ except that the initial and final states are now $\{\phi_i,\phi_f\}$ with $p^2=M_i^2=M_f^2=M^2$, and $F_+^{if}(0)$ is the form factor $f_+^{if}(0)$ multiplied by the appropriate CKM matrix element. Following the existing literature, we split it into two pieces:

$$\Box^{VA}_{\gamma W}(\phi_i,\phi_f,M)=\Box^{VA>}_{\gamma W}+\Box^{VA<}_{\gamma W}(\phi_i,\phi_f,M) \tag{12}$$

which come from the loop integral at $Q^2\equiv -q'^2>Q^2_{\mathrm{cut}}$ and $Q^2<Q^2_{\mathrm{cut}}$ respectively, where $Q^2_{\mathrm{cut}}=2\ \mathrm{GeV}^2$ is a scale above which perturbative QCD works well. The ">" term is flavor- and mass-independent, and was calculated to $\mathcal{O}(\alpha_s^4)$: $\Box^{VA>}_{\gamma W}=2.16\times 10^{-3}$ [40]. In the meantime, direct lattice calculations of the "<" term were performed in two channels [40,41]:

$$\begin{aligned}\Box^{VA<}_{\gamma W}(\pi^+,\pi^0,M_\pi)&=0.67(3)_{\mathrm{lat}}\times 10^{-3}\\ \Box^{VA<}_{\gamma W}(K^0,\pi^-,M_\pi)&=0.28(4)_{\mathrm{lat}}\times 10^{-3}\end{aligned} \tag{13}$$

from which we can also obtain $\Box^{VA<}_{\gamma W}(K^+,\pi^0,M_\pi)=1.06(7)_{\mathrm{lat}}\times 10^{-3}$ through a ChPT matching [38].

The only difference between the integrals in Eq. (10) and (11) is the non-forward (NF) kinematics in the former (i.e. $p\neq p'$ and $p_e\neq 0$), which only affect the integral in the $Q^2<Q^2_{\mathrm{cut}}$ region. Therefore one could similarly split $(\delta f_+)_{\mathfrak{B}}$ into two pieces: $(\delta f_+)_{\mathfrak{B}}=(\delta f_+)^>_{\mathfrak{B}}+(\delta f_+)^<_{\mathfrak{B}}$, where the ">" piece matches trivially to the forward axial γW-box:

$$(\delta f_+)^>_{\mathfrak{B}}=\Box^{VA>}_{\gamma W}f_+(t)\,. \tag{14}$$

On the other hand, the matching between the "<" components is not exact due to the NF effects. We characterize the latter by an energy scale E that could be either M_K-M_π, $(s-M_\pi^2)^{1/2}$ or $(u-M_\pi^2)^{1/2}$. The matching then reads:

$$(\delta f_+)^<_{\mathfrak{B}}=\left\{\Box^{VA<}_{\gamma W}(K,\pi,M_\pi)+\mathcal{O}\left(E^2/\Lambda_\chi^2\right)\right\}f_+(t)\,, \tag{15}$$

where $\mathcal{O}(E^2/\Lambda_\chi^2)$ represents the NF corrections. Numerically, since $E<M_K$, we may multiply the right-hand side of Eq. (15) by M_K^2/Λ_χ^2 as a conservative estimation of the NF uncertainty.

The last virtual correction is the so-called "three-point function" contribution to the charged weak form factors, which was derived within ChPT to $\mathcal{O}(e^2p^2)$ in Ref. [37]. However, it contains an IR-divergent piece that comes from the convection term contribution, and can be resummed to all orders in the chiral expansion by simply adding back the charged weak form factors. This leads to the following partially-resummed ChPT expression:

$$\delta f_{+,3}=(\delta f_+)_{\mathrm{III}}+\left\{(\delta f_{+,3})^{\mathrm{fin}}_{e^2p^2}+\mathcal{O}(e^2p^4)\right\}\,, \tag{16}$$

where the IR-divergent piece $(\delta f_+)_{\mathrm{III}}$ is exact, i.e. resummed to all orders in ChPT. It combines with $(\delta f_+)_{\mathrm{II}}$ in Eq. (9) to give:

$$\mathfrak{Re}\,(\delta f_+)_{\mathrm{II+III}}=\frac{\alpha}{4\pi}\left[-\frac{2}{\beta_i}\tanh^{-1}\beta_i\ln\left(\frac{M_im_e}{M_\gamma^2}\right)+\ln\frac{M_i^2}{M_\gamma^2}-\frac{5}{2}\right]f_+(t)\,, \tag{17}$$

where M_i is the mass of the charged meson (K^+ in K^+_{e3} and π^- in K^0_{e3}) and β_i is the speed of the positron in the rest frame of the charged meson. Meanwhile, the IR-finite pieces, $(\delta f_{+,3})^{\mathrm{fin}}_{e^2p^2}$, are given by the terms in Eqs. (8.3) and (8.5) of Ref. [37] that are not attached to the factor $\ln(M^2/M_\gamma^2)-5/2$, and are subject to $\mathcal{O}(e^2p^4)$ corrections.

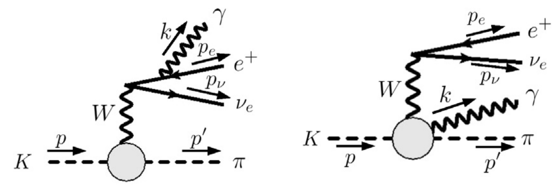

Fig. 3. Bremsstrahlung diagrams.

Next we switch to the bremsstrahlung contributions, as depicted in Fig. 3. Its amplitude is given by:

$$M_{\mathrm{brems}}=-\sqrt{2}G_Fe\bar{u}_{\nu L}\gamma^\mu\left\{\frac{p_e\cdot\varepsilon^*}{p_e\cdot k}+\frac{\not{k}\not{\varepsilon}^*}{2p_e\cdot k}\right\}v_{eL}F_\mu+i\sqrt{2}G_Fe\bar{u}_{\nu L}\gamma^\nu v_{eL}\varepsilon^{\mu *}T_{\mu\nu}(k;p',p)\,, \tag{18}$$

in which the tensor $T^{\mu\nu}$ appears again, except that now it deals with an on-shell photon momentum k whose size is restricted by phase space. Similar to $\delta f_{+,3}$, we find that the most efficient way to calculate the bremsstrahlung contributions is to adopt a partially-resummed ChPT expression for $T^{\mu\nu}$:

$$T^{\mu\nu}=T^{\mu\nu}_{\mathrm{conv}}+\left\{\left(T^{\mu\nu}-T^{\mu\nu}_{\mathrm{conv}}\right)_{p^2}+\mathcal{O}(p^4)\right\}\,, \tag{19}$$

where the full convection term $T^{\mu\nu}_{\mathrm{conv}}$ is explicitly singled out, while the remaining terms in the curly bracket are expanded to $\mathcal{O}(p^2)$. Consequently, one can split M_{brems} into two separately gauge-invariant pieces:

$$M_{\mathrm{brems}}=M_A+M_B\,, \tag{20}$$

where the terms in the curly bracket of Eq. (19) reside in M_B. The contribution to the decay rate from $|M_A|^2$ contains the full IR-divergent structure (which cancels with the virtual corrections), is numerically the largest and does not associate to any chiral expansion uncertainty. The term $2\mathfrak{Re}\left\{M_A^*M_B\right\}+|M_B|^2$, on the other hand, is subject to $\mathcal{O}(e^2p^4)$ corrections. We find that its contribution to $\delta_{K_{e3}}$ is $\lesssim 10^{-3}$, so the associated chiral expansion uncertainty, which is obtained by multiplying the central value with M_K^2/Λ_χ^2, is of the order 10^{-4}.

With the above, we have calculated all EWRC to 10^{-4} and may compare with existing results. The standard parameterization of the fully-inclusive K_{e3} decay rate reads [3]:

$$\Gamma_{K_{e3}}=\frac{G_F^2|V_{us}|^2M_K^5C_K^2}{192\pi^3}S_{\mathrm{EW}}|f_+^{K^0\pi^-}(0)|^2I^{(0)}_{Ke}(\lambda_i)\times(1+\delta^{Ke}_{\mathrm{EM}}+\delta^{K\pi}_{\mathrm{SU(2)}})\,, \tag{21}$$

among which $S_{\mathrm{EW}}=1.0232(3)$ describes the short-distance EWRC [50] (the uncertainty comes from $\delta^{\mathrm{QED}}_{\mathrm{HO}}$ [51]) and $\delta^{Ke}_{\mathrm{EM}}$ describes the long-distance electromagnetic corrections respectively. We also realize that in the existing ChPT treatment a residual component of the electromagnetic corrections, which corresponds exactly to $(\delta f_{+,3})^{\mathrm{fin}}_{e^2p^2}$ in our language, is redistributed into $I^{(0)}_{Ke}(\lambda_i)$ and $\delta^{K\pi}_{\mathrm{SU(2)}}$ that describe the t-dependence of the charged weak form factors and the isospin breaking correction, respectively [32–34]. Therefore, the correspondence between $\delta^{Ke}_{\mathrm{EM}}$ in the ChPT calculation and $\delta_{K_{e3}}$ in our approach reads:

$$\delta^{Ke}_{\mathrm{EM}}=\left(\delta_{K_{e3}}\right)_{\mathrm{tot}}-(S_{\mathrm{EW}}-1)-\left(\delta_{K_{e3}}\right)^{\mathrm{fin}}_3\,. \tag{22}$$

Table 1
Summary of various contributions to $\delta_{K_{e3}}$ (except that from $(\delta f_{+,3})^{\mathrm{fin}}_{e^2p^2}$, see the discussions after Eq. (21)) in units of 10^{-2}.

	from $(\delta f_+)^{\mathrm{fin}}_{\mathfrak{A}}$	from $(\delta f_+)_{\mathfrak{B}}$	from $(\delta f_+)_{\mathrm{I+II+III}}$ and $\lvert M_A\rvert^2$	from $2\mathfrak{Re}\left\{M_A^* M_B\right\}+\lvert M_B\rvert^2$
K^0_{e3}	$-0.09(2)_{\mathrm{sg}}$	$0.49(1)_{\mathrm{lat}}(1)_{\mathrm{NF}}$	$2.97(3)_{\mathrm{HO}}$	$0.12(2)_{e^2p^4}$
K^+_{e3}	$0.96(2)_{\mathrm{sg}}(1)_{\langle r_K^2\rangle}$	$0.64(1)_{\mathrm{lat}}(4)_{\mathrm{NF}}$	$0.97(3)_{\mathrm{HO}}$	$-0.04(1)_{e^2p^4}$

Our results of the different components of $\delta_{K_{e3}}$ are summarized in Table 1, from which we obtain:

$$\delta_{\mathrm{EM}}^{K^+e} = 0.21(2)_{\mathrm{sg}}(1)_{\langle r_K^2\rangle}(1)_{\mathrm{lat}}(4)_{\mathrm{NF}}(1)_{e^2p^4}\times 10^{-2}$$
$$\delta_{\mathrm{EM}}^{K^0e} = 1.16(2)_{\mathrm{sg}}(1)_{\mathrm{lat}}(1)_{\mathrm{NF}}(2)_{e^2p^4}\times 10^{-2}\,. \quad (23)$$

The uncertainties are explained as follows: "sg" is our estimate of the seagull contribution to $I^{\lambda}_{\mathfrak{A}}$, "$\langle r_K^2\rangle$" comes from the experimental uncertainty of the K^+ mean-square charge radius that enters $I^{\lambda}_{\mathfrak{A}}$, "lat" and "NF" are the uncertainties in $(\delta f_+)_{\mathfrak{B}}$ from lattice QCD and the NF effects, respectively, and "e^2p^4" represents the chiral expansion uncertainty in the $2\mathfrak{Re}\left\{M_A^* M_B\right\}+|M_B^2|$ term from the bremsstrahlung contribution. We should compare Eq. (23) to the ChPT result [34]:

$$\left(\delta_{\mathrm{EM}}^{K^+e}\right)_{\mathrm{ChPT}} = 0.10(19)_{e^2p^4}(16)_{\mathrm{LEC}}\times 10^{-2}$$
$$\left(\delta_{\mathrm{EM}}^{K^0e}\right)_{\mathrm{ChPT}} = 0.99(19)_{e^2p^4}(11)_{\mathrm{LEC}}\times 10^{-2}\,. \quad (24)$$

They are consistent within error bars, but Eq. (23) shows a reduction of the total uncertainty by almost an order of magnitude, which can be easily understood as follows. First, in ChPT the $\mathcal{O}(e^2p^4)$ uncertainty is obtained by multiplying the full result, including the IR-singular pieces that are numerically the largest, with M_K^2/Λ_χ^2; meanwhile, within the new formalism those pieces can be evaluated exactly by simply isolating the pole/convection term in $T^{\mu\nu}$ and Γ^{μ}. The remainders are generically an order of magnitude smaller, so their associated $\mathcal{O}(e^2p^4)$ uncertainty is also suppressed. Secondly, in ChPT the LECs $\{X_i\}$ were estimated within resonance models [52,53] and were assigned a 100% uncertainty. On the other hand, some of us pointed out in Ref. [38] that these LECs are associated with the forward axial γW-box diagram, and promoted first-principle calculations with lattice QCD. This effectively transforms the LEC uncertainties in ChPT into the lattice and NF uncertainties in $(\delta f_+)_{\mathfrak{B}}$ which are much better under control.

To conclude, we performed a significantly improved calculation of the EWRC in the K_{e3} channel. We observe no large systematic corrections with respect to previous analyses. Although the error analysis in the $K_{\mu3}$ channel is somewhat more complicated, we deem such large corrections in this channel unlikely. Hence, it is safe to conclude that the EWRC in K_{l3} cannot be responsible for the $K_{\mu2}$–K_{l3} discrepancy in V_{us}. One should then switch to other SM inputs, such as the lattice calculation of $|f_+^{K^0\pi^-}(0)|$ and the theory inputs of $I_{Kl}^{(0)}(\lambda_i)$ and $\delta^{K\pi}_{\mathrm{SU(2)}}$. Finally, our improvement in $\delta^{Ke}_{\mathrm{EM}}$ also opens a new pathway for the precise measurement of V_{us}/V_{ud} through the ratio between the semileptonic kaon and pion decay rate [54]. For instance, we may define:

$$R_V \equiv \frac{\Gamma_{K^0_{e3}}}{\Gamma_{\pi_{e3}}} = 4.035(4)_{\mathrm{PS}}(1)_{\mathrm{RC}}\times 10^8\left|\frac{V_{us}f_+^{K^0\pi^-}(0)}{V_{ud}f_+^{\pi^+\pi^0}(0)}\right|^2 . \quad (25)$$

Since both the RC uncertainties in K^0_{e3} and π_{e3} are now at the 10^{-4} level, the dominant theory uncertainty (apart from lattice inputs) of R_V comes from the K^0_{e3} phase space (PS) integral. We compare this to:

$$R_A \equiv \frac{\Gamma_{K_{\mu2}}}{\Gamma_{\pi_{\mu2}}} = 17.55(3)_{\mathrm{RC}}\left|\frac{V_{us}f_{K^+}}{V_{ud}f_{\pi^+}}\right|^2 \quad (26)$$

which is currently used to extract V_{us}/V_{ud}. We see that R_V possesses a much smaller theoretical uncertainty than R_A, and hence represents a more promising avenue in the future. Our work thus provides a strong motivation for experimentalists to measure the π_{e3} branching ratio with an order-of-magnitude increase in precision [55].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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